

BEYOND THE STANDARD MODEL

Fabio Zwirner

Istituto Nazionale di Fisica Nucleare, Dipartimento di Fisica G. Galilei,
Università di Padova, Via Marzolo n. 8, I-35131 Padova, Italy

Abstract

These lecture notes give a short review of the present theoretical ideas and experimental constraints on possible extensions of the Standard Model, to be used as an invitation to the study of the rich available literature on the subject.

1 A CRITICAL LOOK AT THE STANDARD MODEL

All confirmed experimental data in particle physics are in agreement with the Standard Model (SM) of strong and electroweak interactions. The only ingredient of the SM that has escaped detection so far is the elusive Higgs boson, whose search is ongoing at LEP (the present lower bound on its mass is about 90 GeV [1]) and, if no evidence is found at LEP, will continue at the LHC. The goal of these lectures is to explain that the search for the SM Higgs boson is not the only challenge left for the years to come. There are reasons to believe that some fascinating chapters of the particle physics book, denoted with the generic name of ‘Beyond the Standard Model’ (BSM), have not been disclosed yet, but may become soon (some already are!) accessible to experiment. In the absence of direct and unambiguous experimental information, the discussion of possible BSM physics is subject to strong theoretical prejudice. To enable the reader to share the origin of this prejudice, we begin these lectures with a critical look at the SM, trying to identify its virtues and its unanswered questions. Extensions of the SM are required to answer some of the latter, but they should not spoil the former: as we shall see, this is not an easy task!

1.1 The building blocks of the SM

The theoretical pillar of the SM is local gauge invariance with respect to the gauge group

$$G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y. \quad (1)$$

Gauge invariance completely determines the spin-1 particle content of the SM: the gluons G_μ^A ($A = 1, \dots, 8$), associated with the strong interactions and characterized by the coupling constant g_S ; the bosons W_μ^I ($I = 1, 2, 3$) and B_μ , mediating the electroweak interactions with coupling constants g and g' , respectively, and corresponding, when rearranged into appropriate linear combinations, to the photon γ and to the W^\pm and Z^0 bosons. Gauge invariance also fixes completely the Yang-Mills part of the Lagrangian, including the cubic and quartic self-interactions among the non-abelian gauge bosons, depicted in Fig. 1:

$$\mathcal{L}_{YM} = -\frac{1}{4}G^{\mu\nu A}G_{\mu\nu}^A - \frac{1}{4}W^{\mu\nu I}W_{\mu\nu}^I - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}, \quad (2)$$

where

$$\begin{aligned} G_{\mu\nu}^A &= \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_S f_{ABC} G_\mu^B G_\nu^C, \\ F_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g f_{IJK} W_\mu^J W_\nu^K, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \quad (3)$$

and f_{ABC} and f_{IJK} are the fully antisymmetric $SU(3)$ and $SU(2)$ structure constants, respectively.

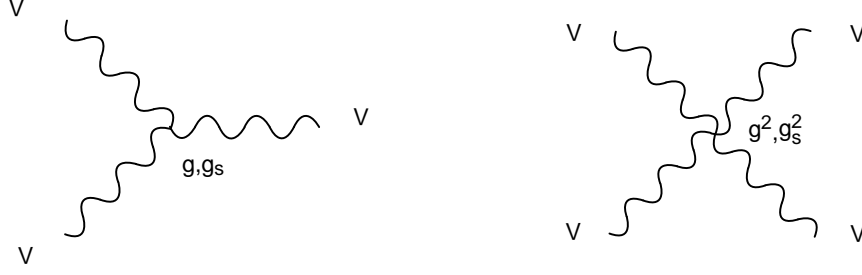


Fig. 1: Pictorial representation of the cubic and quartic self-interactions among the non-abelian gauge bosons of the SM, denoted by the generic symbol V .

The spin- $\frac{1}{2}$ particle content of the SM consists in three generations of quarks and leptons, whose transformation properties under G_{SM} are summarized below

$$q_{aL} \equiv \begin{pmatrix} u_{aL} \\ d_{aL} \end{pmatrix} \sim (3, 2, +1/6), \quad l_{aL} \equiv \begin{pmatrix} \nu_{aL} \\ e_{aL} \end{pmatrix} \sim (1, 2, -1/2), \quad (4)$$

$$u_{aR} \sim (3, 1, +2/3), \quad d_{aR} \sim (3, 1, -1/3), \quad e_{aR} \sim (1, 1, -1).$$

In eq. (4), $a = 1, 2, 3$ is a generation index, and the weak hypercharge Y is normalized according to $Q = T_{3L} + Y$, where Q is the electric charge and T_{3L} the third component of the weak isospin. We have used left- and right-handed chiral projections, defined by $P_{L,R} = (1 \pm \gamma^5)/2$, and $SU(3)_C$ and $SU(2)_L$ indices have been left implicit. Notice the absence of right-handed neutrinos ν_{aR} . Given the quantum number assignments of eq. (4), gauge invariance completely determines the interactions between fermions and gauge bosons, depicted in Fig. 2:

$$\mathcal{L}_F = i\bar{\Psi}\gamma^\mu D_\mu \Psi, \quad (5)$$

where

$$D_\mu = \partial_\mu - ig_S G_\mu^A \lambda^A - ig W_\mu^I \frac{\tau^I}{2} - ig' B_\mu Y. \quad (6)$$

All the fermions are denoted by the collective symbol $\Psi \equiv (q_L, u_R, d_R, l_L, e_R)_{a=1,2,3}$, and the symbols λ^A , $\frac{\tau^I}{2}$ and Y , appearing in the covariant derivative D_μ , stand for the hermitean generators of the different G_{SM} factors in the representation defined by eq. (4).

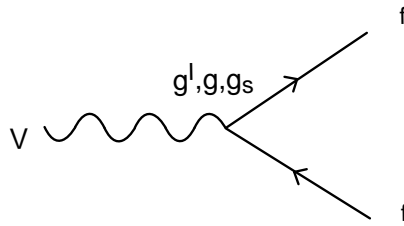


Fig. 2: Pictorial representation of the interactions between the SM fermions, denoted by the generic symbol f , and gauge bosons, denoted by the generic symbol V .

The last but not the least important ingredient of the SM is a complex spin-0 $SU(2)$ -doublet, the so-called Higgs field:

$$\phi \equiv \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \sim (1, 2, +1/2), \quad (7)$$

which appears not only in the part of the Lagrangian containing the spin-0 fields gauge interactions and self-interactions, depicted in Fig. 3:

$$\mathcal{L}_S = (D_\mu \phi)^\dagger (D^\mu \phi) - V, \quad (8)$$

$$V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad (9)$$

but also in the one containing the Yukawa couplings, depicted in Fig. 4:

$$\mathcal{L}_{Yuk} = h_{ab}^{(U)} \overline{q_{aL}} u_{bR} \tilde{\phi} + h_{ab}^{(D)} \overline{q_{aL}} d_{bR} \phi + h_{ab}^{(E)} \overline{l_{aL}} e_{bR} \phi + \text{h.c.}, \quad (10)$$

where for notational convenience we have introduced $\tilde{\phi} \equiv (i\sigma^2 \phi^*) = (\varphi^{0*} \ -\varphi^-)^T \sim (1, 2, -1/2)$, and $h^{(U)}$, $h^{(D)}$ and $h^{(E)}$ are arbitrary 3×3 complex matrices in generation space.

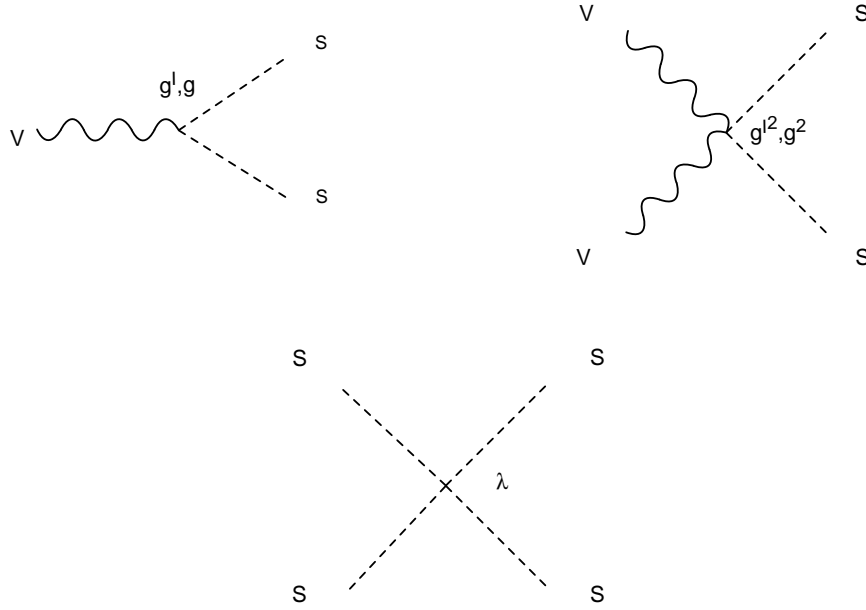


Fig. 3: Pictorial representation of the gauge interactions and self-interactions of the SM spin-0 fields, denoted by the generic symbol S .

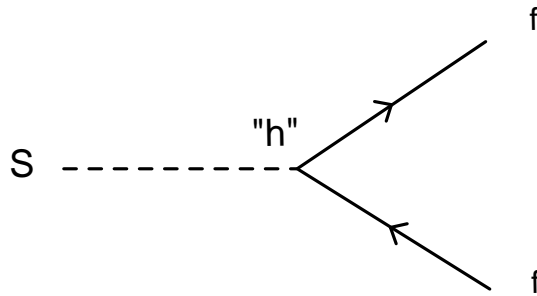


Fig. 4: Pictorial representation of the Yukawa interactions between the SM fermions, denoted by the generic symbol f , and the SM spin-0 fields, denoted by the generic symbol S .

1.2 Chirality, anomaly cancellation, charge quantization

One of the most important properties of the SM is the chirality of its fermion content, which falls into a complex representation of the gauge group. In other words, left- and right-handed fermion fields have different quantum numbers with respect to $SU(2)_L \times U(1)_Y$. In particular, $SU(2)_L \times U(1)_Y$ gauge invariance forbids explicit fermion mass terms of the form $m_f \bar{f}_L f_R + h.c.$, since, for example, left-handed fields are in $SU(2)_L$ doublets and right-handed fields (when present) in $SU(2)_L$ singlets.

The quantum consistency of chiral gauge theories is endangered by the existence of anomalies (for an extensive discussion, see e.g. [2]). We say that a classical symmetry is anomalous when it is not preserved by the quantum corrections. If a gauge symmetry is anomalous, this spoils gauge invariance and/or renormalizability. A well-known criterion for the absence of anomalies is obtained by looking at the triangular graphs with a fermion loop and gauge bosons on the external lines, as in Fig. 5. The

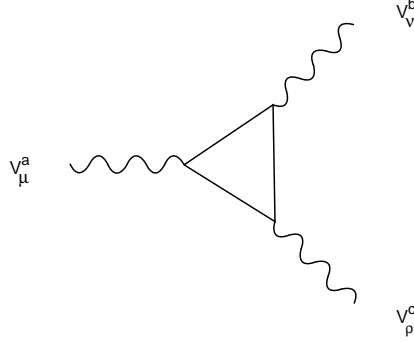


Fig. 5: The triangular graph associated with the anomaly.

contribution of these graphs to the anomaly is proportional to

$$\mathcal{A}^{abc} = \text{tr}_{f_L} \left(\{T^a, T^b\} T^c \right) - \text{tr}_{f_R} \left(\{T^a, T^b\} T^c \right), \quad (11)$$

where T^a are the generators of the gauge group and the two traces run over the group indices of the left-handed and the of right-handed fermions, respectively. It is remarkable that, with the quantum number assignments of eq. (4), in the SM there is an automatic cancellation of all possible gauge anomalies, as the reader can easily verify as an exercise. Seen from another point of view, this sheds light on the phenomenon of charge quantization. At the classical level and within the SM, it is impossible to understand why the electric charges of the different quarks and leptons are related by simple fractional coefficients. On the other hand, it can be shown that asking for the cancellation of gauge anomalies goes a long way towards implying the charge assignments of the SM. Indeed, they are completely fixed if we ask that mixed gauge and gravitational anomalies [3] also cancel and that all charged fermions get a mass. This is a deep aspect of the SM, that may also shed light on the possible fundamental theory unifying gravitational and non-gravitational interactions.

1.3 Spontaneous breaking of the gauge symmetry in the SM

The part of the SM Lagrangian involving the spin-0 field ϕ is instrumental to describe two crucial physical phenomena, to be discussed in turn in the present and in the following subsection. \mathcal{L}_S in eq. (8) is a tool to describe the *spontaneous* breaking of the *local* gauge symmetry $SU(2)_L \times U(1)_Y$ down to $U(1)_Q$, with the associated mass generation for the physical W^\pm and Z^0 bosons.

Choosing $\lambda > 0$ and $\mu^2 < 0$, the classical potential of the SM, eq. (9), is minimized for

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{-\mu^2}{\lambda}}. \quad (12)$$

Correspondingly, non-vanishing masses are generated for the W^\pm and Z^0 bosons

$$m_W^2 = \frac{g^2 v^2}{4}, \quad m_Z^2 = \frac{(g^2 + g'^2) v^2}{4}, \quad (13)$$

whereas the photon remains massless, as dictated by the residual gauge invariance with respect to $U(1)_Q$. Pictorially, one can understand the origin of the gauge boson masses by looking at the second graph of Fig. 3 and by replacing the scalar fields with their constant vacuum expectation values (VEVs).

Remembering the definition of the electroweak mixing angle, $\tan \theta_W \equiv g'/g$, a very important property of the SM is that, before the inclusion of quantum corrections, the following relation holds

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1. \quad (14)$$

For the discussion of possible extensions or modifications of the SM, and also of the quantum corrections within the SM or beyond, it is important to realize that the relation of eq. (14) can be understood in general terms, as a consequence of the so-called *custodial symmetry* [4]. On general grounds, imposing only invariance under $U(1)_Q$, the most general mass Lagrangian for the electroweak vector bosons is:

$$\mathcal{L}_m = \frac{1}{2} m_W^2 (W_{1\mu} W_1^\mu + W_{2\mu} W_2^\mu) + \frac{1}{2} (W_{3\mu} B_\mu) \begin{pmatrix} M^2 & M'^2 \\ M'^2 & M''^2 \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}, \quad (15)$$

where the masslessness of the photon implies:

$$M^2 M''^2 = M'^4, \quad M^2 + M''^2 = m_Z^2. \quad (16)$$

Then the mass matrix is fully determined by one parameter only, for example M^2 , and it is easy to show that $\rho = m_W^2/M^2$, so that $\rho = 1$ is equivalent to $M^2 = m_W^2$. Consider now the classical SM potential, eq. (9). The crucial observation is that V is invariant under a global symmetry group that is larger than the $SU(2)_L \times U(1)_Y$ gauge group. Indeed, if we define

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}, \quad (17)$$

and we observe that $\phi^\dagger \phi = (\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2)/2$, we can immediately realize that V , being function of $\phi^\dagger \phi$ only, is invariant under $O(4) \sim SU(2) \times SU(2)$. To clarify the physical interpretation, we can introduce the matrix

$$\Phi \equiv \begin{pmatrix} \tilde{\phi} & \phi \end{pmatrix} \equiv \begin{pmatrix} \varphi^{0*} & \varphi^+ \\ -\varphi^- & \varphi^0 \end{pmatrix}, \quad (18)$$

and observe that $\phi^\dagger \phi = (1/2) \text{tr} (\Phi^\dagger \Phi)$. We can then immediately identify the action of the $SU(2)_L \times SU(2)_R$ group that leaves V invariant:

$$\Phi \longrightarrow U_L \Phi U_R^\dagger = e^{-i\vec{\sigma} \cdot \vec{\omega}_L} \Phi e^{i\vec{\sigma} \cdot \vec{\omega}_R}. \quad (19)$$

The other parts of the SM Lagrangian are not all invariant under such a global symmetry. \mathcal{L}_S would be invariant, were it not for the fact that ϕ and $\tilde{\phi}$ have opposite weak hypercharges, so it becomes invariant in the limit where $g' \rightarrow 0$. The Yukawa Lagrangian involving a given fermion doublet, say the one for the third generation quarks, would be invariant if we assign the right-handed quarks to a doublet of $SU(2)_R$ and we take the limit of equal Yukawa couplings for the top and bottom quarks, corresponding to $M_t = M_b$. Anyway, the global $SU(2)_L \times SU(2)_R$ invariance is spontaneously broken on the vacuum. Observing that

$$\langle \Phi \rangle \propto \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}, \quad (20)$$

it is immediate to see that on the vacuum $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$, where the ‘diagonal’ or ‘vectorial’ $SU(2)_V$ corresponds to $\vec{\omega}_L = \vec{\omega}_R$. Since the $SU(2)_L$ vector bosons transform as a triplet under $SU(2)_V$, the custodial symmetry implies $M = m_W$ and therefore $\rho = 1$. Incidentally, this explains why in the SM the largest corrections to the ρ parameter are controlled by the $M_t - M_b$ mass difference (the effects proportional to g' are numerically much smaller).

For a modern discussion of any BSM physics, an important information is the fact that the SM description of the spontaneous breaking of the electroweak gauge symmetry has been tested to an impressive level of precision at LEP, at the Tevatron and in other experiments at lower energies. Many observable quantities that are sensitive to the SM radiative corrections and, potentially, also to BSM ones, have been measured with high accuracy. The picture that emerges is summarized in Fig. 6 [5], and is in excellent agreement with the SM predictions. Now that the top quark mass has been directly mea-

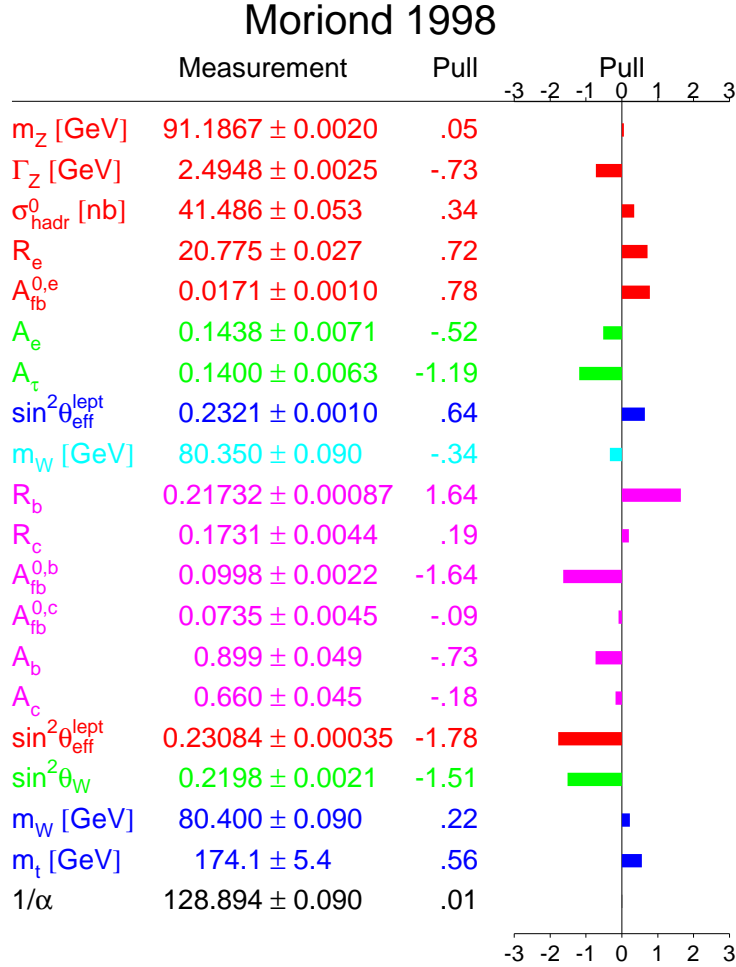


Fig. 6: Results of a recent fit to precision electroweak observables, compared with the corresponding SM predictions.

sured with good precision by the Tevatron experiments, the data are sufficiently precise to be sensitive to the mild, logarithmic dependence of the SM radiative corrections on the Higgs boson mass m_H , and favour values of m_H close to the present experimental upper bound, as can be seen in Fig. 7 [5]. The most important message of electroweak precision tests, however, concerns possible BSM physics: only very delicate deviations from the SM predictions are allowed. This is a very strong constraint on theorists’ imagination, and allows to discard several extensions or modifications of the SM proposed in the past. For an updated review of electroweak precision tests and of their interpretation within and beyond the SM, see e.g. [6].

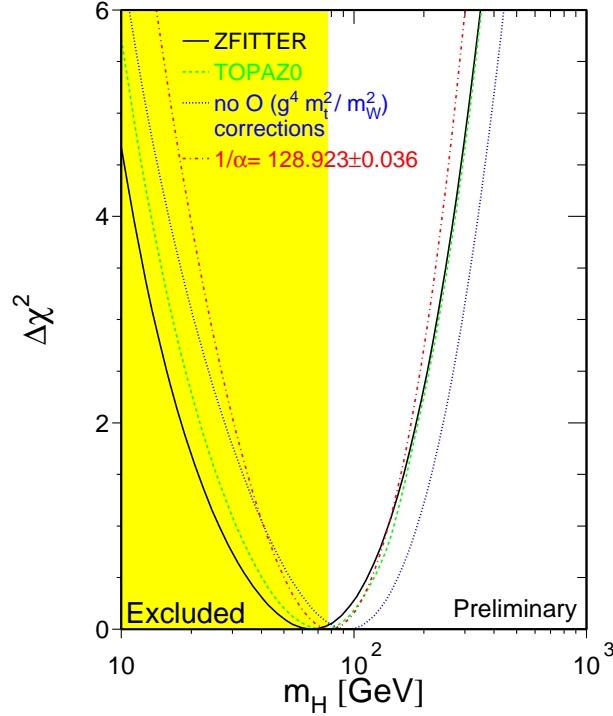


Fig. 7: Results of a fit of the electroweak precision data to the Higgs boson mass m_H within the SM.

1.4 Explicit breaking of the flavour symmetry in the SM

The second important physical phenomenon where the SM spin-0 field plays a crucial rôle is the *explicit* breaking of the *global* flavour symmetry, via the Yukawa couplings of eq. (10). In the absence of \mathcal{L}_{Yuk} , the SM Lagrangian has a huge $[U(3)]^5$ global symmetry, corresponding to unitary transformations in generation space for the five irreducible fermionic representations of the gauge group, eq. (4). Indeed, since the $U(1)_Y$ subgroup is gauged, the additional global symmetry is more precisely $[SU(3)]^5 \times [U(1)]^4$, where the four $U(1)$ factors correspond to the total baryon number, B , and to the individual lepton numbers, (L_e, L_μ, L_τ) . The flavour symmetry implies that gauge interactions do not distinguish among the three generations of quarks and leptons. In the real world, this symmetry must be broken, since we observe a complicated pattern of masses, mixing angles and phases for the SM fermions. The rôle of \mathcal{L}_{Yuk} in the SM is precisely the explicit breaking of the flavour symmetry down to the $[U(1)]^4$ associated with (B, L_e, L_μ, L_τ) , which correspond to accidental global symmetries of the SM, in agreement with the experimental bounds on baryon- and lepton-number non-conserving processes.

As a side remark, we observe that B and $L \equiv L_e + L_\mu + L_\tau$ are anomalous global symmetries, as the reader can easily check as an exercise. This means that they can be violated at the quantum level by non-perturbative effects. Only the combination $(B - L)$ is an exact, non-anomalous symmetry of the SM. In accelerator experiments, this fact does not have very important implications, since the rates for the corresponding processes are exponentially suppressed. On the other hand, in the thermal history of our universe these interactions could have been unsuppressed at temperatures of the order of the electroweak scale. This has prompted many speculations about the possibility of generating the cosmological matter-antimatter asymmetry by physics at the electroweak scale. Qualitatively, the SM may have all the necessary ingredients for such a phenomenon (B non-conservation, C and CP violation, departure from thermal equilibrium during the electroweak phase transition). At present, it seems difficult to obtain a quantitatively satisfactory description of baryogenesis within the SM framework. However, better realizations of this idea may be possible within simple extensions of the SM. For recent reviews and references on electroweak baryogenesis, see e.g. [7].

Coming back to the explicit breaking of the flavour symmetry, the Yukawa part of the SM Lagrangian realizes it in a very special way. On the one hand, the breaking is very strong, as one can realize by staring at the observed pattern [8] of the 9 fermion masses and of the 4 parameters (3 mixing angles and 1 CP-violating phase) appearing in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. On the other hand, the *only* source of flavour violation in the SM Lagrangian is precisely the CKM matrix, controlling the weak charged-current interactions. In the SM, all the tree-level flavour-changing-neutral-current (FCNC) couplings, i.e. those of the photon, of the Z^0 and of the physical Higgs boson, are flavour-diagonal. FCNC processes are induced only by loop effects, controlled by the CKM matrix and sufficiently suppressed to guarantee agreement with experimental data on flavour physics.

To conclude this subsection, we recall some important constraints from flavour physics [8] that are passed with flying colours by the SM, but are very severe censors of its possible modifications. As for ‘quark flavour’,

$$\Delta m_K = (3.491 \pm 0.009) \times 10^{-12} \text{ MeV}, \quad (21)$$

$$|\epsilon_K| = (2.28 \pm 0.01) \times 10^{-3}, \quad (22)$$

$$\Delta m_{B_d} = (3.05 \pm 0.12) \times 10^{-13} \text{ GeV}, \quad (23)$$

$$BR(B \rightarrow X_s \gamma) = (2.32 \pm 0.67) \times 10^{-4}, \quad (24)$$

$$d_n < 9 \times 10^{-26} \text{ e cm}, \quad (25)$$

..., with a meaning of the symbols that should be obvious to the particle physicists in the audience. As for ‘lepton flavour’,

$$BR(\mu \rightarrow e \gamma) < 4.9 \times 10^{-11}, \quad (26)$$

$$BR(K_L \rightarrow \mu e) < 3.3 \times 10^{-11}, \quad (27)$$

$$d_e < 4 \times 10^{-27} \text{ e cm}, \quad (28)$$

... Finally, as for total baryon and lepton number,

$$t_{1/2}^{(\beta\beta)_{0\nu}} > 5.6 \times 10^{24} \text{ yrs}, \quad (29)$$

$$\tau(p \rightarrow e^+ \pi^0) > 6.8 \times 10^{32} \text{ yrs}, \quad (30)$$

$$\tau(p \rightarrow K^+ \nu) > 1 \times 10^{32} \text{ yrs}, \quad (31)$$

... As we shall see, all these constraints can be violated if we go BSM!

1.5 The SM as an effective theory and its problems

Despite the remarkable achievements described in the previous subsections, no physicist believes that the SM is really the ultimate theory of elementary particles, since, among the other things, it has about 20 arbitrary parameters, which may seem too many for a fundamental theory, and it leaves several unanswered questions, for example some concerning *unification* and *flavour*. The unification problem is related to the gauge interactions, whose pattern of groups and representations is complicated and arbitrary. Why should there be three different factors in the gauge group, with the associated coupling constants taking the values they do? Why should the fermions transform according to such an odd choice of chiral representations of $SU(2)_L \times U(1)_Y$, so that parity is violated in weak interactions? The flavour problem has to do with the Yukawa interactions of the SM, which introduce several arbitrary parameters into the model. There is no explanation for the existence of three fermion generations with the same gauge quantum numbers, nor for the complicated observed pattern of masses, mixing angles and phases.

The previous arguments suggest to go beyond the SM, but there is more: it is quite obvious that the SM *must* be extended! Among the ‘hard’ arguments supporting the previous statement, the strongest one is the fact that the SM does not include a quantum theory of gravitational interactions. Immediately

after comes the fact that some of the SM couplings are not asymptotically free, making it almost surely inconsistent as a formal Quantum Field Theory.

This does not give us direct information on the form of the required SM extensions, but brings along an important conceptual implication: the SM should be seen as an *effective field theory* [9], valid up to some physical cut-off scale Λ . Assuming that the SM correctly identifies the degrees of freedom at the electroweak scale (this may not be true, for example, in the case of the SM Higgs field), the basic rule of the game is to write down the most general local Lagrangian compatible with the SM symmetries [i.e. the $SU(3) \times SU(2) \times U(1)$ gauge symmetry and the Poincaré symmetry], scaling all dimensionful couplings by appropriate powers of Λ . The resulting dimensionless coefficients are then to be interpreted as parameters, which can be either fitted to experimental data or (if one is able to do so) theoretically determined from the fundamental theory replacing the SM at the scale Λ . Very schematically (and omitting all coefficients and indices, as well as many theoretical subtleties):

$$\begin{aligned}
\mathcal{L}_{eff} = & \Lambda^4 + \Lambda^2 \Phi^2 \\
& + (D\Phi)^2 + \bar{\Psi} \not{D} \Psi + F^{\mu\nu} F_{\mu\nu} + F^{\mu\nu} \tilde{F}_{\mu\nu} + \bar{\Psi} \Psi \Phi + \Phi^4 \\
& + \frac{\bar{\Psi} \Psi \Phi^2}{\Lambda} + \frac{\bar{\Psi} \sigma^{\mu\nu} \Psi F_{\mu\nu}}{\Lambda} \\
& + \frac{\bar{\Psi} \Psi \bar{\Psi} \Psi}{\Lambda^2} + \frac{\Phi^2 F^{\mu\nu} F_{\mu\nu}}{\Lambda^2} \\
& + \dots,
\end{aligned} \tag{32}$$

where Ψ stands for the generic quark or lepton field, Φ for the SM Higgs field, F for the field strength of the SM gauge fields, and D for the gauge-covariant derivative. The first line of eq. (32) contains two operators carrying positive powers of Λ , a cosmological constant term, proportional to Λ^4 , and a scalar mass term, proportional to Λ^2 . Barring for the moment the discussion of the cosmological constant term, which becomes relevant only when the model is coupled to gravity, it is important to observe that *no* quantum SM symmetry is recovered by setting to zero the coefficient of the scalar mass term. On the contrary, the SM gauge invariance forbids fermion mass terms of the form $\Lambda \bar{\Psi} \Psi$. The second line of eq. (32) contains operators with no power-like dependence on Λ , but only a milder, logarithmic dependence, due to infrared renormalization effects between the cut-off scale Λ and the electroweak scale. The operators of dimension $d \leq 4$ exhibit two remarkable properties: all those allowed by the symmetries are actually present in the SM; both baryon number and the individual lepton numbers are automatically conserved. The third and fourth line of eq. (32) are the starting point of an expansion in inverse powers of Λ , containing infinitely many terms. For energies and field VEVs much smaller than Λ , the effects of these operators are suppressed, and the physically most interesting ones are those that violate some accidental symmetries of the $d \leq 4$ operators. For example, as we shall see, a $d = 5$ operator of the form $\bar{\Psi} \Psi \Phi^2$ can generate a L-violating Majorana neutrino mass of order G_F^{-1}/Λ (where $G_F^{-1/2} \simeq 300$ GeV is the Fermi scale); some of the $d = 6$ four-fermion operators can be associated with flavour-changing neutral currents (FCNC) or with baryon- and lepton-number-violating processes such as proton decay, and so on.

At this point, a question naturally emerges: where is the cut-off scale Λ , at which the expansion of eq. (32) loses validity and the SM must be replaced by a more fundamental theory? Two extreme but plausible answers can be given:

- (I) Λ is not much below the Planck scale, $M_P \equiv G_N^{-1/2}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV, as roughly suggested by the measured strength of the fundamental interactions, including the gravitational ones.
- (II) Λ is not much above the Fermi scale, as suggested by the idea that new physics must be associated with electroweak symmetry breaking.

In the absence of an explicit realization at a fundamental level, each of the above answers can be heavily criticized. The criticism of **(I)** has to do with the existence of the ‘quadratically divergent’ scalar mass operator, which becomes more and more ‘unnatural’ as Λ increases above the electroweak scale [10]. On general theoretical grounds, we would expect for such operator a coefficient of order 1, but experimentally we need a strongly suppressed coefficient, of order G_F^{-1}/Λ^2 . However, after taking into account quantum corrections, this coefficient can be conceptually decomposed into the sum of two separate contributions, controlled by the physics below and above the cut-off scale, respectively. Answer **(I)** would then require a subtle (malicious?) conspiracy between low-energy and high-energy physics, ensuring the desired fine-tuning. The criticism of **(II)** has to do instead with the $d > 4$ operators: in order to sufficiently suppress the coefficients of the dangerous operators associated with proton decay, FCNC, etc., the new physics at the cut-off scale Λ must have quite non-trivial properties! On purely dimensional grounds, the effective operators associated with electroweak precision tests would suggest $\Lambda \gtrsim 10^3$ GeV, those associated with FCNC would suggest $\Lambda \gtrsim 10^6$ GeV, those associated with proton decay would suggest $\Lambda \gtrsim 10^{15}$ GeV, and so on for many other examples (however, we should keep in mind that the coefficients of these operators may be suppressed by loop factors or by symmetries of the underlying fundamental theory). As we shall see, this is a potential problem also for the extensions of the SM discussed in the present lectures.

At the moment, answer **(I)** is not very popular in the physics community, since we do not have the slightest idea on how the required conspiracy could possibly work at the fundamental level. Conceptually, such a possibility can be theoretically tested in an ultraviolet-finite Theory of Everything: as daring as it may sound, with the advent and the continuing development of string theories and their generalizations, we may not be very far from the implementation of the first quantitative tests. More concretely, such a possibility can be experimentally tested in the near future, via the search for the Higgs boson at LEP, at the Tevatron and at the LHC. A clear picture of the implications of **(I)** is given in Fig. 8, which shows, for various possible choices of Λ in the SM, the values of the top quark and Higgs boson masses allowed by the following two requirements [11]:

- The SM effective potential should not develop, besides the minimum corresponding to the experimental value of the electroweak scale, other minima with lower energy and much larger value of the Higgs field. In first approximation, this amounts to requiring the SM effective Higgs self-coupling, $\lambda(Q)$, not to become negative at any scale $Q < \Lambda$: for a given value of the top quark mass M_t , this sets a lower bound on the SM Higgs mass m_H .
- The SM effective Higgs self-coupling should not develop a Landau pole at scales smaller than Λ : for a given value of M_t , this sets an upper bound on m_H . Such constraint has a meaning which goes beyond perturbation theory, as suggested by the infrared structure of the SM renormalization group equation for $\lambda(Q)$ and confirmed by explicit lattice computations [12].

Fig. 8 includes some recent refinements [13] of the original analysis, such as two-loop renormalization group equations, optimal scale choice, finite corrections to the pole top and Higgs masses, etc. For very large cut-off scales, $\Lambda = 10^{16} - 10^{19}$ GeV, the results are quite stable and can be summarized as follows: for a top quark mass close to 175 GeV, as measured at the Tevatron collider [8], the only allowed range for the SM Higgs mass is $130 \text{ GeV} \lesssim m_H \lesssim 200 \text{ GeV}$. This means that, even in the absence of a direct discovery of new physics beyond the SM, answer **(I)** could be falsified by LEP, the Tevatron and the LHC in two possible ways: either by discovering a SM-like Higgs boson lighter than 130 GeV, or by excluding a SM-like Higgs boson in the 130–200 GeV range!

Answer **(II)**, instead, gives rise to a well-known conceptual bifurcation:

- (IIa)** In the description of electroweak symmetry breaking, the elementary SM Higgs scalar is replaced by some fermion condensate, induced by a new strong interaction near the Fermi scale. This

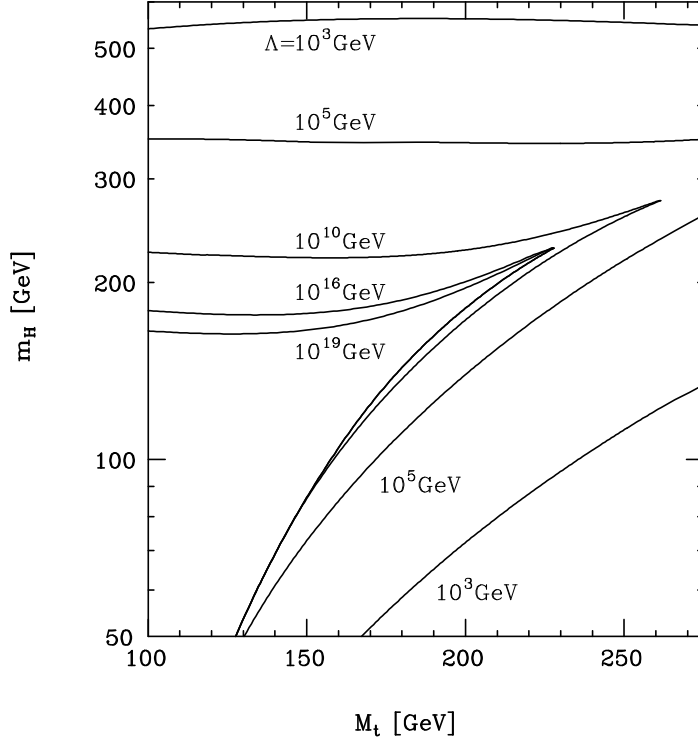


Fig. 8: Bounds in the (M_t, m_H) plane, for various choices of Λ .

includes old and more recent variants of the so-called *technicolor* models (‘extended’, ‘walking’, ...), to be discussed in Sect. 2.

- (IIIb) The SM is embedded in a model with broken supersymmetry, and supersymmetry-breaking mass splittings between the SM particles and their superpartners are of the order of the electroweak scale. This approach, generically denoted as *low-energy supersymmetry*, will be discussed extensively in Sect. 5.

To understand better the motivations for new physics near the electroweak scale, we take now a more concrete look at the naturalness problem. Such problem arises whenever we insist, as in the SM, on the presence of an elementary Higgs field in the lagrangian to describe the breaking of the electroweak symmetry, and we want to extrapolate the model to a scale Λ much larger than the Fermi scale. The tree-level potential of the SM is characterized by a mass parameter μ^2 and by a dimensionless quartic coupling λ . One combination of these two parameters, essentially μ^2/λ , is fixed by fitting the VEV v of the SM Higgs field to the measured value of the Fermi constant, defining the scale of electroweak symmetry breaking. The squared mass m_H^2 of the physical Higgs particle, proportional to μ^2 , or, equivalently, to λv^2 , is instead a free parameter of the SM. While the lower bound on the Higgs mass comes from experiment, arguments based on perturbative unitarity and triviality suggest that self-consistency of the SM is broken unless $m_H < \mathcal{O}(1 \text{ TeV})$. This is hard to reconcile, from the effective field theory point of view, with the fact that, already at one-loop, there are quadratically divergent contributions to the Higgs boson mass, as can be checked by performing an explicit calculation with a naive cut-off regularization in momentum space. The question then arises: how can the Higgs boson mass be of the order of the electroweak scale and not of the order of the physical ultraviolet cutoff of the theory?

The problem outlined above is generic for theories containing elementary spin-0 fields. For ex-

ample, consider a model with a complex spin-0 field of mass m_B and a two-component fermion of mass m_F , with a Yukawa coupling λ_F and a quartic scalar coupling λ_B . The one-loop corrections to the boson mass include two quadratically divergent contributions of opposite sign, one involving a fermion loop and controlled by the Yukawa coupling λ_F , the other one involving a scalar loop and controlled by the four-point coupling λ_B , and have the form

$$\delta m_B^2 \propto (\lambda_B - \lambda_F^2) \Lambda^2 + \dots, \quad (33)$$

where Λ is the ultraviolet cutoff, the minus sign comes from the fermion loop, and the dots stand for less divergent terms. The situation is radically different in the case of the loop corrections to the fermion mass, the latter being protected by a chiral symmetry in the limit $m_F \rightarrow 0$. The one-loop diagram correcting the fermion mass is logarithmically divergent and proportional to the fermion mass, giving

$$\delta m_F \propto \lambda_F^2 m_F. \quad (34)$$

Therefore, the fermion mass can be naturally small. In the case of the scalar mass, what we need to make it naturally small is a symmetry relating bosons and fermions, and enforcing the vanishing of the coefficient of Λ^2 in (33), not only at one loop but also at higher orders: the only known candidate is supersymmetry. Alternatively, we need to dispose of elementary spin-0 fields.

To conclude this section, we should mention two other naturalness problems of the SM, in some sense analogous to the one discussed for the mass term of the elementary spin-0 field: the strong-CP problem (for reviews and references, see e.g. [14]) and the cosmological constant problem (for reviews and references, see e.g. [15]). The first is related to the fact that the SM symmetries do not forbid a CP-violating lagrangian term of the form

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} G^{\mu\nu A} \tilde{G}_{\mu\nu}^A, \quad (35)$$

where $\tilde{G}_{\mu\nu}^A \equiv (1/2)\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma A}$. The parameter θ is constrained by the experimental bounds on the neutron electric dipole moment:

$$d_n^{exp} \lesssim 10^{-25} \text{ e cm} \quad \Rightarrow \quad |\theta| \lesssim 2 \times 10^{-10}. \quad (36)$$

The second originates from the fact that a constant contribution to the vacuum energy, such as the Λ^4 term in eq. (32), is also allowed by the SM symmetries, and becomes physically relevant when the SM is coupled to gravity. From the present observations on the expansion rate of the universe, we can derive the following bound on the cosmological constant term:

$$\Lambda_{cosm} \lesssim \frac{G_F^{-1}}{M_P} \sim 10^{-4} \text{ eV}. \quad (37)$$

From the above equation, it is immediate to realize that the cosmological constant problem is by far the toughest naturalness problem in fundamental physics. There is no time here to discuss the solutions to these problems proposed so far: the curious reader is encouraged to consult the excellent reviews mentioned above.

2 ‘DYNAMICAL’ ELECTROWEAK SYMMETRY BREAKING

Since, as we have seen in Sect. 1, the naturalness problem of the SM is due to its Higgs sector, the most conservative strategy in the search for a solution is to get rid of elementary spin-0 fields in the lagrangian, replacing the Higgs doublet with some dynamical fermion condensate, induced by some new strong interaction. This is the approach of the so-called technicolor models and their generalizations (for reviews and references, see e.g. [16]).

We begin by describing the simplest technicolor (TC) model [17]. Imagine that, in addition to the SM gauge interactions, there is a gauge group factor $SU(N)_{TC}$, with coupling constant g_{TC} , that is asymptotically free and whose dynamical scale Λ_{TC} , analogous to Λ_{QCD} in the SM, is just above the electroweak scale. Imagine also that, besides the SM fermions, singlets under $SU(N)_{TC}$, there is also one generation of massless ‘technifermions’, corresponding to the following anomaly-free representation of $SU(N)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$Q_L \equiv \begin{pmatrix} U \\ D \end{pmatrix}_L \sim (N, 1, 2, 0), \quad U_R \sim (N, 1, 1, +1/2), \quad D_R \sim (N, 1, 1, -1/2). \quad (38)$$

In analogy with QCD, the kinetic terms for the technifermions have a chiral $SU(2)_L \times SU(2)_R$ symmetry, and, as the QCD interactions induce the formation of a condensate in the spin-0, isospin-0 channel, $\langle \bar{u}_L u_R + \bar{d}_L d_R \rangle \sim \Lambda_{QCD}^3$, the TC interactions cause the formation of a condensate $\langle \bar{U}_L U_R + \bar{D}_L D_R \rangle \sim \Lambda_{TC}^3$. The global $SU(2)_L \times SU(2)_R$ chiral symmetry is dynamically broken to the diagonal $SU(2)_V$, the only difference with QCD being that the scale $f_\pi \sim 100$ MeV is replaced by a much higher scale F_{TC} . The goldstone bosons of the broken chiral symmetry, analogous to the pions of QCD, are called technipions. Since the condensate also breaks $SU(2)_L \times U(1)_Y$ down to $U(1)_Q$, the technipions provide the longitudinal degrees of freedom of the massive W^\pm and Z^0 bosons, whose masses are given by:

$$m_W = \frac{g F_{TC}}{2}, \quad m_Z = \frac{\sqrt{g^2 + g'^2} F_{TC}}{2}. \quad (39)$$

Notice that the custodial symmetry introduced in the previous section is at work in guaranteeing $\rho = 1$. We then see that, in order to reproduce the measured values of m_W and m_Z , it must be $F_{TC} \sim 250$ GeV.

Such a dynamical description of the spontaneous breaking of the gauge symmetry is very elegant, and has interesting analogies with the BCS explanation of superconductivity, but the model is too simple to be realistic, and an important feature of the SM is missing: there is no breaking of the flavour symmetry, and fermion masses and mixing are not reproduced. From this point of view, the SM description with an elementary Higgs has an advantage, since the interactions of the elementary Higgs allow to kill two birds with a stone.

To avoid this problem, extended technicolor (ETC) models have been proposed [18]. In these models, technifermions are allowed to talk to fermions by introducing another, larger gauge group, the ETC group G_{ETC} , spontaneously broken at a certain scale $\Lambda_{ETC} > \Lambda_{TC}$ down to $SU(N)_{TC} \times G_{SM}$. In this case, both ordinary fermions and technifermions feel the ETC interactions, and loop diagrams involving the exchange of the ETC gauge bosons and of technifermions can generate mass terms for the ordinary fermions. Concentrating for definiteness on quarks, the typical size of the mass terms is

$$m_q \sim \frac{g_{ETC}^2 \langle \bar{Q}_L Q_R \rangle_{\Lambda_{ETC}}}{\Lambda_{ETC}^2}. \quad (40)$$

A naive scaling argument suggests $\langle \bar{Q}_L Q_R \rangle_{\Lambda_{ETC}} \sim \Lambda_{TC}^3$. Then giving mass to the heavy SM fermions would require Λ_{ETC} to be very close to Λ_{TC} . However, to reproduce the observed quark mixing, the ETC interactions must also connect different generations: this immediately leads to phenomenological problems with FCNC, since the typical coefficients of the induced $d = 6$ four-fermion operators would scale as $g_{ETC}^2 / \Lambda_{ETC}^2$. To avoid these problems in a natural way, we should have $\Lambda_{ETC} \gtrsim 500$ TeV. The existence of heavy fermions and the observed suppression of FCNC are incompatible in ETC models, whereas they are miraculously accommodated in the SM.

To avoid the problems of the naive, QCD-like ETC models, models of ‘walking’ technicolor were suggested [19]. These models try to loosen the link between fermion masses and FCNC by arranging for $\langle \bar{Q}_L Q_R \rangle_{\Lambda_{ETC}} \gg \Lambda_{TC}^3$. This may occur if there is a non-conventional dynamical behaviour of the theory at high scales, which does not follow the QCD paradigm. In this is true, one may gain up to a

factor $\Lambda_{ETC}/\Lambda_{TC}$ in the theoretical expression for the fermion masses, eq. (40). However, this is still insufficient to explain the observed value of the top quark mass.

Attempts to solve the top mass problem in technicolor models have been made. For example, in ‘topcolor-assisted’ technicolor [20], one introduces an additional strong interaction to generate the large observed value of M_t . These models have some similarity with previous ‘top-condensate’ models [21], where the condensate breaking the electroweak gauge symmetry is not made of techniquarks but of the top quarks themselves, but are claimed to avoid some of the phenomenological problems of the latter.

We should also mention some other generic phenomenological problems of TC-ETC models and of their variations:

- Typically, these models have large global symmetries, with associated pseudo-goldstone bosons, and one must make sure that the corresponding phenomenological constraints are respected
- In all these models, one should also explain dynamically the breaking of the ETC symmetry, and this may be as difficult as explaining dynamically the breaking of the electroweak gauge symmetry
- All these models typically contain a large number of particles with $SU(2) \times U(1)$ quantum numbers: it is in general problematic to reconcile this feature with the present electroweak precision data. For some crucial observables, the typical deviations from the SM predictions are much larger than the present experimental accuracy [6]

In conclusion, dynamical electroweak symmetry breaking is an attractive idea, still looking for a satisfactory model. Even if the present experimental data do not seem to push in this direction (but we should not forget that the SM Higgs boson has not been found yet!), we should keep an eye on this possibility and analyze present and future experimental data without prejudice. In the absence of a definite, self-consistent and convincing model, it may be wise to study this option with theoretical tools that are as model-independent as possible. The most suitable ones seem to be the phenomenological effective lagrangians reviewed in [22]. With these tools, we can test triple gauge boson couplings at LEP2 and at the Tevatron, and the scattering of longitudinal vector bosons at the LHC: the experimental study of possible deviations from the SM predictions may shed light on a possible strongly-interacting dynamics at work in the symmetry-breaking sector of the SM.

3 GRAND-UNIFIED THEORIES AND PROTON DECAY

The basic idea of grand unification is that the gauge interactions as observed at the presently accessible energies, with the different numerical values of their coupling constants, are just the remnants of a theory with a single gauge coupling constant, spontaneously broken at a very high scale. The simplest possibility is to have a single scale $M_U \gg m_Z$, at which a simple gauge group G is spontaneously broken down to the SM gauge group:

$$\begin{array}{ccc} M_U & & m_Z \\ G & \longrightarrow & G_{SM} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_C \times U(1)_Q . \\ g_U & & (g_S, g, g') \end{array} \quad (41)$$

There is a vast literature on grand unification, both with and without supersymmetry, and many excellent reviews are available (see e.g. [23]). We shall limit ourselves here to a qualitative overview of its most relevant features.

We may ask what are the candidate groups G for a grand-unified theory (GUT). First of all, G must be simple, in order to allow for a single gauge coupling constant, and of rank $r \geq 4$, in order to contain G_{SM} as a subgroup. Second, G must allow for complex but anomaly-free representations, in which we should embed the SM quarks and leptons. Incidentally, we recall that in the context of GUTs (and of supersymmetric models) it is customary to work with a basis of purely left-handed fermions,

exploiting the fact that $(u^c)_L \leftrightarrow (u_R)^c$, $(d^c)_L \leftrightarrow (d_R)^c$, $(e^c)_L \leftrightarrow (e_R)^c$, where the superscript c denotes charge conjugation. Taking the general classification of simple Lie algebras, the simplest solutions are $SU(5)$ at $r = 4$, $SO(10)$ at $r = 5$ and E_6 at $r = 6$.

The simplest realization of the grand-unification idea is the minimal, non-supersymmetric $SU(5)$ model of Georgi and Glashow [24] (for a previous attempt with partial unification, see [25]). The gauge bosons of such model belong to the adjoint representation of the rank-4 simple group $SU(5)$, 24_V : besides the SM gauge bosons, there are 12 additional ones, $(X, Y) \sim (\bar{3}, 2, +5/6)$ and their conjugates (\bar{X}, \bar{Y}) , of mass M_V . These bosons have fractional electric charge and carry both baryon and lepton number, $\Delta B = \Delta L = \pm 1$. Each fermion generation is arranged in an anti-fundamental representation, $\bar{5}_F$, and in the antisymmetric product of two fundamentals, 10_F . In terms of SM fermions, the two representations decompose as follows:

$$\bar{5}_F \rightarrow (d^c, l), \quad 10_F \rightarrow (q, u^c, e^c). \quad (42)$$

Notice that in minimal $SU(5)$, as in the SM, there is no need to introduce a right-handed neutrino, represented by a left-handed antineutrino ν^c in the present conventions. The scalar fields introduced to describe the different stages of spontaneous symmetry breaking correspond to an adjoint representation, 24_S , containing 12 Goldstone bosons and 12 additional scalars of mass M_Σ , and an anti-fundamental representation, $\bar{5}_S$, containing the SM Higgs boson and an additional triplet $H \sim (\bar{3}, 1, +1/3)$ of mass M_H .

The first stage of symmetry breaking is controlled by the VEV of the 24_S , of order M_U . The masses M_V, M_Σ, M_H have model-dependent relations with M_U , but in first approximation we can assume that they are all of order M_U . The breaking of the SM gauge group at the electroweak scale is controlled instead by the VEV of the SM Higgs doublet contained in the $\bar{5}_S$. The fermions get masses via their Yukawa couplings, of the form

$$h^{(10)} \cdot 10_F \times 10_F \times 5_S, \quad h^{(5)} \cdot \bar{5}_F \times 10_F \times \bar{5}_S, \quad (43)$$

where generation indices have been understood. These Yukawa couplings cannot give rise to a realistic pattern of fermion masses and mixing (even if some predictions such as the M_b/M_τ ratio [26] are intriguingly close to being correct), but are chosen to keep the model simple.

Non-minimal grand-unified models can be constructed, by enlarging one or more of the following: the gauge group, the fermion content, the scalar content. They will not be discussed here.

One of the most dramatic phenomenological implications of grand-unification is the possibility of $\Delta B = \Delta L = \pm 1$ nucleon decay, for example $p \rightarrow e^+ \pi^0$. There are two types of tree-level Feynman diagrams, involving three quarks and a lepton on the external lines, that could induce such a process. The first type involves the exchange of virtual (X, Y) vector bosons on an internal line, and the corresponding rate scales as $\Gamma \sim g_U^4/M_V^4$; the second type involves the exchange of the scalar Higgs triplet H , and the corresponding rate scales as $\Gamma \sim h^4/M_H^4$, where h is a Yukawa coupling. In the case of gauge-mediated nucleon decay, the amount of model-dependence is small. In first approximation, from the experimental bound [8] $\tau_p^{exp}(p \rightarrow e^+ \pi^0) > 6.8 \times 10^{32} \text{ yrs}$, and from the approximate formula $\tau_p^{th}(p \rightarrow e^+ \pi^0) \sim 10^{28 \pm 1} \text{ yrs} \cdot [M_V(\text{GeV})/2 \times 10^{14}]^4$, we can deduce a stringent lower bound on the grand-unification scale M_U .

The important point is that, from the measured values of two of the low-energy gauge couplings, we can extract a rather precise prediction for g_U , M_U and the third low-energy gauge coupling. In first approximation, we can just solve the one-loop renormalization group equations (RGEs) for the running gauge couplings [27],

$$\frac{dg_A^2}{dt} = \frac{b_A}{8\pi^2} g_A^4, \quad (A = 1, 2, 3; t = \log Q), \quad (44)$$

with the boundary conditions

$$g_3(M) = g_2(M) = g_1(M) \equiv g_U, \quad (45)$$

where $g_3 \equiv g_S$, $g_2 \equiv g$ and $g_1 \equiv \sqrt{5/3}g'$. In the absence of new physics thresholds between M_U and the scale $Q \ll M_U$, the RGEs are trivially solved by

$$\frac{1}{g_A^2(Q)} = \frac{1}{g_U^2} + \frac{b_A}{8\pi^2} \log \frac{M_U}{Q}. \quad (46)$$

The one-loop beta function coefficients appropriate for the SM are easily computed [27]:

$$b_3^{SM} = -7, \quad b_2^{SM} = -\frac{19}{6}, \quad b_1^{SM} = \frac{41}{10}. \quad (47)$$

Starting from three input data at the electroweak scale $Q = m_Z$ [8], for example α_3 , α_{em} and $\sin^2 \theta_W$, we can perform consistency checks of the grand-unification hypothesis in different models.

In the minimal $SU(5)$ model [24], and indeed in any other model where eq. (45) holds and the light-particle content is just the SM one (with no intermediate mass scales between m_Z and M_U), eqs. (46) and (47) are incompatible with experimental data. This was first realized by noticing that the prediction $M_U \simeq 10^{14-15}$ GeV, obtained by using as inputs α_3 and α_{em} , is incompatible with the limits on nucleon decay. Subsequently, also the prediction $\sin^2 \theta_W \simeq 0.21$ was shown to be in conflict with experimental data [28], and this conflict became more and more significant with the progressive accumulation of high-quality data from the LEP and Tevatron experiments. We shall see in Sect. 5 how grand-unification can be phenomenologically more successful when combined with supersymmetry.

4 NEUTRINO MASSES AND OSCILLATIONS

In the SM, the only independent neutrino fields are the left-handed neutrinos ν_L , belonging to the doublets $l_L \equiv (\nu_L, e_L)^T \sim (1, 2, -1/2)$ and carrying lepton number $L = +1$. By CP, they are associated to right-handed antineutrinos $(\nu^c)_R = (\nu_L)^c \equiv C\bar{\nu}_L^T$ (C is the charge conjugation matrix), carrying lepton number $L = -1$. Neutrino are strictly massless in the SM, taken as a renormalizable theory: gauge invariance and the absence of a right-handed neutrino forbid both explicit mass terms and mass terms induced by the Yukawa couplings to the Higgs field. However, as we shall now discuss, neutrino masses can arise in many extensions of the SM, or even by simply allowing for the presence of non-renormalizable operators in the SM, taken as an effective theory. Before moving to such a discussion, we anticipate that many extensions of the SM, for example grand-unified models based on the $SO(10)$ gauge group, contain among their fermion fields an additional right-handed neutrino $\nu_R \sim (1, 1, 0)$ for each generation, transforming as a singlet under the SM gauge group, equivalent via CP to a left-handed antineutrino $(\nu^c)_L = (\nu_R)^c = C\bar{\nu}_R^T$, and conventionally carrying lepton number $L = +1$.

In the presence of both left-handed and right-handed neutrino fields, and exploiting the fact that neutrinos are electrically neutral, different types of neutrino mass terms can be considered. Dirac mass terms are of the form:

$$m_D(\bar{\nu}_L \nu_R + h.c.). \quad (48)$$

They conserve the total lepton number ($\Delta L = 0$), and are analogous to the mass terms for the charged fermions. Because of $SU(2)_L \times U(1)_Y$ invariance, these terms must arise via some Yukawa coupling to a doublet Higgs field with non-vanishing VEV, for example the SM Higgs field. Majorana mass terms for left-handed neutrinos are of the form:

$$\frac{m_L}{2}[\bar{\nu}_L(\nu^c)_R + h.c.]. \quad (49)$$

They violate lepton number by two units ($\Delta L = \pm 2$), and have no analogue for the electrically charged fermions. These mass terms have the quantum number of a Higgs triplet, so they may arise either in the presence of an extended Higgs sector, or from effective non-renormalizable interactions. Majorana mass terms for right-handed neutrinos are of the form:

$$\frac{m_R}{2}[(\nu^c)_L \nu_R + h.c.]. \quad (50)$$

They also violate lepton number by two units ($\Delta L = \pm 2$), and have no analogue for the electrically charged fermions. The crucial feature of these mass terms is that they are invariant under the SM gauge group, so they can explicitly appear in the lagrangian.

To conclude this brief discussion of neutrino masses, let us see in slightly more detail how the different neutrino mass terms may originate in different models.

To get Dirac mass terms, it is sufficient to add right-handed neutrinos ν_{aR} (we are now reintroducing generation indices, omitted for simplicity in the previous discussion) to the SM fermion content. In this case, we can add to the gauge-invariant Yukawa couplings of the SM the following ones

$$h_{ab}^{(N)} \overline{l_{aL}} \nu_{bR} \tilde{\phi} + \text{h.c.} \quad (51)$$

The diagonalization of the lepton masses proceeds as in the quark sector, and the leptonic charged currents, expressed in the basis of the fermion mass eigenstates, involve a mixing matrix analogous to the CKM matrix of the quark sector. The partial lepton numbers (L_e, L_μ, L_τ) are thus violated by the charged current weak interactions, whereas the total lepton number $L = L_e + L_\mu + L_\tau$ is still conserved. The weak point of this theoretical framework is the absence of any understanding of the smallness of the neutrino masses (the present upper bounds are [8] $m_{\nu_e} \lesssim 5$ eV, $m_{\nu_\mu} < 160$ keV, $m_{\nu_\tau} < 23$ MeV) with respect to the masses of the charged fermions.

The simplest way to obtain Majorana mass terms for the left-handed neutrinos is to keep the fermion and Higgs content of the SM, and to add to the renormalizable interactions of the SM an effective $d = 5$ non-renormalizable interaction of the form

$$\frac{\lambda}{M} \overline{(l_L)^c} \sigma^I l_L \phi^T \sigma^I \phi + \text{h.c.} \quad (52)$$

If the dimensionless coupling λ is in the perturbative regime and the mass scale M is larger than the electroweak scale, putting the SM Higgs fields equal to their VEVs generates a left-handed Majorana mass term of order $m_\nu \sim \lambda v^2/M \ll v$, thus naturally small. Such a possibility can arise in GUTs, and in particular in left-right symmetric GUTs such as those based on the $SO(10)$ gauge group, where a right-handed neutrino is automatically included in the fermion spectrum and a large mass scale M can be associated to some stage of the spontaneous breaking of the grand-unified gauge group.

The previous considerations lead us to a rather elegant scheme that may explain the smallness of the neutrino masses and the absence of right-handed neutrinos from the theory at the electroweak scale, the so-called see-saw mechanism [29]. If the neutrino mass matrix has the following structure

$$\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}, \quad (53)$$

with the hierarchy $m_L \ll m_D \ll m_R$, then the spectrum of the theory contains, for each generation, a light neutrino of mass $m_{\text{light}} \sim m_D^2/m_R$, essentially left-handed, and a heavy neutrino of mass $m_{\text{heavy}} \sim m_R$, essentially right-handed. We can make connection with the effective theory point of view by drawing the Feynman diagram associated with the left-handed Majorana mass term: in the fundamental theory, it is a four-point interaction involving two Higgs fields and two left-handed neutrinos, induced by the tree-level exchange of a heavy right-handed neutrino and controlled by the Yukawa couplings associated with the Dirac neutrino mass terms. Integrating out the heavy right-handed neutrinos, and putting the SM Higgs fields at their VEVs, we generate precisely the effective interaction of eq. (52).

The strongest constraints on Majorana neutrino masses come from neutrinoless double beta decay, $(\beta\beta)_{0\nu}$ for short, corresponding to $(Z, A) \rightarrow (Z + 2, A) + 2e^-$, which violates lepton number by two units. From the present experimental limits [8] on the decays of ${}^{76}\text{Ge}$, corresponding to $t_{1/2}^{(\beta\beta)_{0\nu}} > 5.6 \times 10^{24}$ yrs, we can deduce that the appropriate combination of Majorana neutrino masses entering the theoretical expression for the decay rate must be smaller than roughly 1 eV.

Another important phenomenon connected with the possibility of neutrino masses are neutrino oscillations. These have been extensively discussed in many reviews [30], thus I refer the reader to the dedicated literature for the study of this very important topic, which is likely to give us the first convincing evidence for BSM physics!

5 LOW-ENERGY SUPERSYMMETRY: THE MSSM

We have already mentioned the hierarchy or naturalness problem of the SM as the main motivation for low-energy supersymmetry. Before starting the discussion of supersymmetric extensions of the SM, it is appropriate to recall that there are other theoretical motivations for supersymmetry:

- it is the most general symmetry of the S-matrix consistent with a non-trivial relativistic quantum field theory;
- it is an interesting laboratory for the analytical study of the non-perturbative regime of non-trivial four-dimensional quantum field theories;
- it seems to play an important rôle for the consistency of superstrings, candidate unified theories of all interactions, including the gravitational ones.

However, only the naturalness problem requires the existence of supersymmetric particles with masses within the TeV scale, making low-energy supersymmetry testable at present and forthcoming colliders, and a suitable subject for this School.

5.1 Generalities on supersymmetric lagrangians

The formulation and the perturbative properties of supersymmetric field theories are described in many excellent textbooks and reviews (see e.g. [31]). Here we summarize, in a non-technical way, the main ingredients that play a rôle in the construction of supersymmetric extensions of the SM at the electroweak scale. The non-expert reader is urged to consult the pedagogical literature on this subject for a systematic and self-contained presentation.

Supersymmetric field theories [32] are based on the supersymmetry algebra [33], an extension of the Poincaré algebra, obtained from the latter by adding some generators of fermionic character, obeying anticommutation relations. We limit ourselves here to the case of simple ($N = 1$) supersymmetry in $d = 4$ space-time dimensions. Most realistic models are based on this case, which allows for matter fields transforming in chiral representations of the gauge group. Realistic models with extended ($N > 1$) supersymmetry are more difficult to construct and will not be discussed here, even if their special field-theoretical properties may justify dedicated investigations. The fundamental anticommutation relation of the $N = 1$ supersymmetry algebra is:

$$\{Q_\alpha, Q_\beta\} = -2(\gamma^\mu C)_{\alpha\beta} P_\mu, \quad (54)$$

where C is the charge conjugation matrix and Q is a Majorana spinor, commuting both with the generators P_μ of space-time translations and with the generators T^a of possible (global and/or local) internal symmetries. This implies that particles sitting in the same irreducible representations of supersymmetry have spins differing by $1/2$, but the same internal quantum numbers and, as long as supersymmetry is unbroken, the same mass.

To construct supersymmetric lagrangians, it is convenient to start from the irreducible representations of supersymmetry, or supermultiplets. Leaving aside all technicalities, we recall that the two types of supermultiplets used in the construction of globally supersymmetric extensions of the SM are the chiral and the vector supermultiplets. Chiral supermultiplets contain a complex spin-0 field φ , a Majorana spinor ψ (carrying the same degrees of freedom of a left-handed Weyl spinor ψ_L) and a complex

scalar F , corresponding to an auxiliary non-propagating field. Vector superfields contain (in the so-called Wess-Zumino gauge) a real spin-1 field V_μ , a Majorana spinor λ and a real scalar auxiliary field D .

With the previous superfields, there is a definite rule to construct the most general supersymmetric, gauge invariant, renormalizable lagrangian. In the case of a simple gauge group G , to which we associate the hermitean generators T^a and the gauge coupling constant g , the result is, after elimination of the auxiliary fields via their equations of motion:

$$\begin{aligned} \mathcal{L}_{SUSY} = & -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \frac{i}{2}\bar{\lambda}^a \gamma^\mu D_\mu \lambda^a + (D^\mu \varphi)_i^\dagger (D_\mu \varphi)^i + \frac{i}{2}\bar{\psi}_i \gamma^\mu D_\mu \psi^i \\ & + \left[i\sqrt{2}g\bar{\psi}_i \lambda^a (T^a \varphi)^i + \text{h.c.} \right] - \frac{1}{2} \left[\frac{\partial^2 w}{\partial \varphi^i \partial \varphi^j} \bar{\psi}^i \psi^j + \text{h.c.} \right] - V(\varphi, \varphi^\dagger), \end{aligned} \quad (55)$$

where the scalar potential reads

$$V = F_i^* F^i + \frac{g^2}{2} D^a D^a = \sum_i \left| \frac{\partial w}{\partial \varphi^i} \right|^2 + \sum_a \frac{g^2}{2} \left[\varphi_i^* (T^a)^i_j \varphi^j \right]^2 \geq 0, \quad (56)$$

and $w(\phi)$ is a gauge-invariant polynomial of degree three in the fields φ^i , called superpotential.

Notice how supersymmetry brings along a unification of couplings. In ordinary theories, such as the SM, one may introduce three different types of dimensionless couplings: gauge couplings, Yukawa couplings and quartic scalar couplings. Supersymmetric theories allow only for two different types of couplings, gauge couplings and superpotential couplings, and the dimensionless couplings appearing in the scalar potential are related to these.

One of the main features of supersymmetric theories is their milder ultraviolet behaviour, summarized by the so-called ‘non-renormalization theorems’ [34]. For example, there is no independent renormalization of the superpotential parameters at any finite order in perturbation theory. A related property is the absence of field-dependent quadratic divergences, as long as there are no anomalous $U(1)$ factors in the gauge group. We shall now use this property to give an intuitive explanation of how supersymmetry may help [35] in the solution of the naturalness problem of the SM.

Another way of looking at the naturalness problem of the SM is to consider its one-loop effective potential, which contains a quadratically divergent contribution proportional to

$$\text{Str } \mathcal{M}^2(\varphi) \equiv \sum_i (-1)^{2J_i} (2J_i + 1) m_i^2(\varphi), \quad (57)$$

where the sum is over the various field-dependent mass eigenvalues $m_i^2(\varphi)$, with weights accounting for the number of degrees of freedom and the statistics of particles of different spin J_i . In the SM, $\text{Str } \mathcal{M}^2$ depends on the Higgs field, and induces a quadratically divergent contribution to the Higgs squared mass, already identified as the source of the naturalness problem. A possible solution of the problem may be provided by $N = 1$ global supersymmetry. For unbroken $N = 1$ global supersymmetry, $\text{Str } \mathcal{M}^2$ is identically vanishing, due to the fermion-boson degeneracy within supersymmetric multiplets. The vanishing of $\text{Str } \mathcal{M}^2$ persists if global supersymmetry is spontaneously broken and there are no anomalous $U(1)$ factors [36]. Indeed, to solve the naturalness problem of the SM one could allow for harmless, field-independent quadratically divergent contribution to the effective potential: this is actually used to classify the so-called soft supersymmetry-breaking terms [37], to be discussed later. With typical mass splittings Δm within the MSSM supermultiplets, the field-dependent logarithmic divergences in the effective action induce corrections to the Higgs mass parameter which are at most $O(\Delta m^2)$: the hierarchy is then stable if $\Delta m \lesssim 1$ TeV.

5.2 The Minimal Supersymmetric Standard Model (MSSM)

We shall now describe the construction of the MSSM lagrangian (for reviews, see e.g. [38, 39]). We begin by identifying the minimal renormalizable lagrangian with global $N = 1$ supersymmetry that extends the SM one [40].

If we keep $G \equiv SU(3)_C \times SU(2)_L \times U(1)_Y$ as the gauge group, the spin-1 fields of the SM are just replaced by vector superfields. The theory contains then some new spin- $\frac{1}{2}$ Majorana particles, called ‘gauginos’: the $SU(3)$ ‘gluinos’ \tilde{g} , the $SU(2)$ ‘winos’ \tilde{W} , and the $U(1)$ ‘bino’ \tilde{B} .

Similarly, the spin- $\frac{1}{2}$ matter fields of the SM are replaced by the corresponding chiral superfields, including, as new degrees of freedom, a complex spin-0 field for each quark or lepton chirality state: the ‘squarks’ $\tilde{q}_L \equiv (\tilde{u}_L, \tilde{d}_L)^T$, \tilde{u}_R, \tilde{d}_R and the ‘sleptons’ $\tilde{l}_L \equiv (\tilde{\nu}_L, \tilde{e}_L)^T$, \tilde{e}_R , in three generations as their fermionic superpartners. Remembering that chiral superfields contain left-handed spinors, for each generation we shall introduce the superfields Q, L, U^c, D^c and E^c , whose fermionic components are $q_L, l_L, (u^c)_L, (d^c)_L$ and $(e^c)_L$, respectively.

Finally, we must introduce additional multiplets containing the spin-0 degrees of freedom necessary for the Higgs mechanism. To give masses to all quarks and leptons, to cancel gauge anomalies and to avoid a massless fermion of charge ± 1 , we must introduce at least two Higgs doublet chiral supermultiplets

$$H_1 \equiv \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \sim (1, 2, -1/2), \quad H_2 \equiv \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \sim (1, 2, +1/2). \quad (58)$$

They contain, in addition to the spin-0 fields (H_1^0, H_1^-) and (H_2^+, H_2^0) , denoted here with the same symbols of the corresponding superfields without any risk of confusion, also the associated spinor fields $(\tilde{H}_1^0, \tilde{H}_1^-)$ and $(\tilde{H}_2^+, \tilde{H}_2^0)$, the so-called ‘higgsinos’.

With the chiral superfields introduced above, the most general gauge invariant and renormalizable superpotential is

$$\begin{aligned} w = & h^U Q U^c H_2 + h^D Q D^c H_1 + h^E L E^c H_1 + \mu H_1 H_2 \\ & + \lambda Q D^c L + \lambda' L L E^c + \mu' L H_2 \\ & + \lambda'' U^c D^c D^c. \end{aligned} \quad (59)$$

In the previous formula, generation indices are understood, but we should keep in mind that the couplings $\mu', (h^U, h^D, h^E)$ and $(\lambda, \lambda', \lambda'')$ are tensors with one, two and three generation indices, respectively. The first line of eq. (59) contains only terms which conserve the total baryon and lepton numbers, B and L , whereas the terms in the second line obey the selection rule $\Delta B = 0, |\Delta L| = 1$, and the ones in the third line $\Delta L = 0, |\Delta B| = 1$. The simultaneous presence of the terms in the second and in the third line would be phenomenologically unacceptable: for example, there could be superfast proton decay mediated by the exchange of a squark.

The usual way out from this phenomenological embarrassment is the assumption of a discrete, multiplicative symmetry called R-parity, defined as

$$R = (-1)^{2S+3B+L}, \quad (60)$$

where S is the spin quantum number. In practice, the R-parity assignments are $R = +1$ for all ordinary particles (quarks, leptons, gauge and Higgs bosons), $R = -1$ for their supersymmetric partners (squarks, sleptons, gauginos and higgsinos).

The choice of the gauge group and of the chiral superfield content, and the requirement of an exact R-parity, are enough to specify the form of the globally supersymmetric lagrangian \mathcal{L}_{SUSY} which extends the SM one. However, this cannot be the whole story: we know that supersymmetry is broken in Nature, since we do not observe, for example, scalar partners of the electron degenerate in mass with it.

The problem of supersymmetry breaking will be briefly mentioned later on in these lectures. To parametrize the phenomenology at the electroweak scale, the MSSM Lagrangian is obtained [41] by adding to $\mathcal{L}_{\text{SUSY}}$ a collection $\mathcal{L}_{\text{SOFT}}$ of explicit but *soft* supersymmetry-breaking terms, which preserve the good ultraviolet properties of supersymmetric theories. In general, $\mathcal{L}_{\text{SOFT}}$ contains [37] mass terms for scalar fields and gauginos, as well as a restricted set of scalar interaction terms proportional to the corresponding superpotential couplings

$$-\mathcal{L}_{\text{SOFT}} = \sum_i \tilde{m}_i^2 |\varphi^i|^2 + \frac{1}{2} \sum_A M_A \bar{\lambda}^A \lambda^A + \left(h^U A^U Q U^c H_2 + h^D A^D Q D^c H_1 + h^E A^E L E^c H_1 + m_3^2 H_1 H_2 + \text{h.c.} \right), \quad (61)$$

where φ^i ($i = H_1, H_2, Q, U^c, D^c, L, E^c$) denotes the generic spin-0 field, and λ_A ($A = 1, 2, 3$) the generic gaugino field. Observe that, since A^U, A^D and A^E are matrices in generation space, the most general form of $\mathcal{L}_{\text{SOFT}}$ contains in principle a huge number of free parameters. Moreover, as will be discussed later, for generic values of these parameters there can be serious phenomenological problems with flavour-changing neutral currents and with new sources of CP-violation. For now, we shall ignore intergenerational mixing.

5.2.1 The MSSM spectrum

The tree-level scalar potential associated with the MSSM Lagrangian,

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{SOFT}}, \quad (62)$$

is a function of all the spin-0 fields of the model. To discuss $SU(2)_L \times U(1)_Y$ gauge symmetry breaking, it is usually assumed that all squark and slepton fields have vanishing VEVs, and the attention is restricted to the Higgs potential:

$$V_0 = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 H_2 + \text{h.c.}) + \frac{g^2}{8} \left(H_2^\dagger \vec{\sigma} H_2 + H_1^\dagger \vec{\sigma} H_1 \right)^2 + \frac{g'^2}{8} \left(|H_2|^2 - |H_1|^2 \right)^2, \quad (63)$$

where

$$m_1^2 \equiv \mu^2 + m_{H_1}^2, \quad m_2^2 \equiv \mu^2 + m_{H_2}^2, \quad (64)$$

and, thanks to the possibility of redefining the phases of the Higgs superfields, it is not restrictive to assume that $m_3^2 < 0$, so that the potential is minimized for

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad v_1, v_2 \in \mathbb{R}^+. \quad (65)$$

For the potential to be bounded from below, we have to require that

$$\mathcal{S} \equiv m_1^2 + m_2^2 - 2|m_3^2| \geq 0. \quad (66)$$

In order to get non-vanishing VEVs at the minimum, we must destabilize the origin in field space:

$$\mathcal{B} \equiv m_1^2 m_2^2 - m_3^4 \leq 0. \quad (67)$$

To minimize the potential, it is convenient to use the auxiliary variables

$$v^2 \equiv v_1^2 + v_2^2, \quad \tan \beta \equiv \frac{v_2}{v_1}, \quad (68)$$

so that the minimization conditions assume the simple form

$$\sin 2\beta = \frac{-2m_3^2}{m_1^2 + m_2^2}, \quad v^2 = \frac{4}{g^2 + g'^2} \frac{m_1^2 - m_2^2 \tan \beta}{\tan^2 \beta - 1}. \quad (69)$$

With these expressions in our hands, we are now ready to study the MSSM spectrum.

The R-even sector of the MSSM contains, to begin with, all the spin-1 and spin- $\frac{1}{2}$ particles of the SM. The only difference is the fact that the mass terms for gauge bosons and fermions are now originated by two independent VEVs. For example, the tree-level expressions for the W and Z masses are

$$m_W^2 = \frac{g^2}{2}(v_1^2 + v_2^2), \quad m_Z^2 = \frac{g^2 + g'^2}{2}(v_1^2 + v_2^2). \quad (70)$$

Quarks of charge $Q = 2/3$ have tree-level masses proportional to v_2 , quarks of charge $Q = -1/3$ and charged leptons have tree-level masses proportional to v_1 . Neglecting for the moment intergenerational mixing, and considering for example the third generation,

$$M_t^2 = h_t^2 v_2^2, \quad M_b^2 = h_b^2 v_1^2, \quad M_\tau^2 = h_\tau^2 v_1^2, \quad (71)$$

where (h_t, h_b, h_τ) are dimensionless Yukawa couplings.

A non-trivial structure arises in the Higgs boson sector, where we have, to begin with, two complex doublets, H_1 and H_2 , amounting to eight real degrees of freedom. After shifting the fields according to

$$H_1 = \begin{pmatrix} v_1 + \frac{S_1 + iP_1}{\sqrt{2}} \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} \frac{H_2^+}{v_2 + \frac{S_1 + iP_1}{\sqrt{2}}} \\ H_2^- \end{pmatrix}, \quad (72)$$

and after decoupling the Goldstone bosons, $G^0 = -\cos \beta P_1 + \sin \beta P_2$, $G^+ = -\cos \beta (H_1^-)^* + \sin \beta H_2^+$, $G^- = (G^+)^*$, we are left with five physical degrees of freedom. Two of them correspond to a charged (complex) field,

$$H^+ = \sin \beta (H_1^-)^* + \cos \beta (H_2^+), \quad H^- = (H^+)^*, \quad (73)$$

with tree-level mass

$$m_{H^\pm}^2 = m_W^2 + m_A^2, \quad (74)$$

where

$$m_A^2 = -m_3^2 \left(\tan \beta + \frac{1}{\tan \beta} \right). \quad (75)$$

The remaining degrees of freedom correspond to three neutral states. One of them is CP-odd,

$$A^0 = \sin \beta P_1 + \cos \beta P_2, \quad (76)$$

with mass m_A^2 as in eq. (75). The other two are CP-even, and the corresponding mass eigenstates and eigenvalues are obtained diagonalizing the mass matrix for S_1 and S_2 :

$$\mathcal{M}_S^2 = \begin{pmatrix} m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta & -(m_Z^2 + m_A^2) \sin \beta \cos \beta \\ -(m_Z^2 + m_A^2) \sin \beta \cos \beta & m_Z^2 \sin^2 \beta + m_A^2 \cos^2 \beta \end{pmatrix}. \quad (77)$$

The explicit expression for the mass eigenvalues is trivially obtained,

$$m_{h,H}^2 = \frac{1}{2} \left[(m_A^2 + m_Z^2) \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta} \right], \quad (78)$$

and the corresponding mass eigenstates read, in order of increasing mass,

$$h = -\sin \alpha S_1 + \cos \alpha S_2, \quad H = \cos \alpha S_1 + \sin \alpha S_2, \quad (79)$$

where the mixing angle α is conventionally chosen such that $-\frac{\pi}{2} \leq \alpha \leq 0$ and is given by

$$\cos 2\alpha = -\cos 2\beta \left(\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2} \right), \quad \sin 2\alpha = -\sin 2\beta \left(\frac{m_H^2 + m_h^2}{m_H^2 - m_h^2} \right). \quad (80)$$

It is important to notice the tree-level mass relations

$$m_{H^\pm}^2 = m_W^2 + m_A^2, \quad (81)$$

$$m_h^2 + m_H^2 = m_Z^2 + m_A^2, \quad (82)$$

which imply

$$m_{H^\pm} > m_W, \quad m_H > m_Z, \quad m_A > m_h, \quad m_h < m_Z |\cos 2\beta| < m_Z. \quad (83)$$

It is also important to realize that, at tree level, all Higgs masses and couplings can be expressed in terms of two parameters only: for example, we can choose as independent parameters $(m_A, \tan \beta)$, or $(m_h, \tan \beta)$, or (m_h, m_A) . Some more details on the phenomenology of the MSSM Higgs sector will be given later in these lectures.

We now move to the spectrum of the R-odd sector of the MSSM.

The spin-0 s-particles are the superpartners of the ordinary quarks and leptons. Even neglecting inter-generational mixing, there is another kind of mixing that has to be taken into account. Barring the case of sneutrinos, for which the corresponding fermion is purely left-handed, the spin-0 partners of left- and right-handed quark and leptons can in general mix, and their mixing is described by 2×2 matrices of the form

$$\mathcal{M}_f^2 = \begin{pmatrix} m_{\tilde{f}_{LL}}^2 & m_{\tilde{f}_{LR}}^2 \\ m_{\tilde{f}_{LR}}^2 & m_{\tilde{f}_{RR}}^2 \end{pmatrix}, \quad (f = e, u, d), \quad (84)$$

where

$$m_{\tilde{f}_{LL}}^2 = m_{\tilde{f}_L}^2 (soft) + m_{\tilde{f}_L}^2 (D-term) + m_f^2, \quad (85)$$

$$m_{\tilde{f}_{RR}}^2 = m_{\tilde{f}_R}^2 (soft) + m_{\tilde{f}_R}^2 (D-term) + m_f^2,$$

$$m_{\tilde{f}_{LR}}^2 = \begin{cases} m_f (A_f + \mu \tan \beta) & f = e, \mu, \tau, d, s, b \\ m_f (A_f + \mu \cot \beta) & f = u, c, t \end{cases} \quad (86)$$

and the D-term contribution is given by

$$m^2 (D-term) = m_Z^2 \cos 2\beta (T_{3L} - \sin^2 \theta_W Q). \quad (87)$$

In general, therefore, one expects the interaction eigenstates, $(\tilde{f}_L, \tilde{f}_R)$, to differ from the mass eigenstates, $(\tilde{f}_1, \tilde{f}_2)$ in order of increasing mass. However, the amount of L-R mixing is proportional to the mass of the corresponding fermion, and is usually negligible for the first two generations.

Among the spin- $\frac{1}{2}$ sparticles, we find the strongly interacting gluinos, \tilde{g} , which do not mix with other states and whose mass is an independent parameter of \mathcal{L}_{SOFT} .

The weakly interacting spin- $\frac{1}{2}$ sparticles are two charged and four neutral gaugino-Higgsino mixtures, usually called “charginos” and “neutralinos”, respectively.

The two chargino mass eigenstates, $(\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm)$ in order of increasing mass, are superpositions of winos \tilde{W}^\pm and Higgsinos $\tilde{H}_{2,1}^\pm$, and their mixing is described by the mass Lagrangian:

$$\mathcal{L}_{mass}^{CHA} = -\frac{1}{2} \begin{pmatrix} \tilde{W}^+ & \tilde{H}_2^+ & \tilde{W}^- & \tilde{H}_1^- \end{pmatrix} \begin{pmatrix} 0 & \mathcal{M}_C^T \\ \mathcal{M}_C & 0 \end{pmatrix} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \\ \tilde{W}^- \\ \tilde{H}_1^- \end{pmatrix} + \text{h.c.}, \quad (88)$$

where the 2×2 mass matrix \mathcal{M}_C is given by

$$\begin{pmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & \mu \end{pmatrix}, \quad (89)$$

and is diagonalized by the bi-unitary transformation

$$U^* \mathcal{M}_C V^\dagger = \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix}. \quad (90)$$

Similarly, the mixing between the four neutralino states is described by the mass lagrangian

$$\mathcal{L}_{mass}^{NEU} = -\frac{1}{2} (\tilde{\Psi}^0)^T \mathcal{M}_N \tilde{\Psi}^0 + \text{h.c.}, \quad (91)$$

where $(\tilde{\Psi}^0)^T \equiv (\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ and the 4×4 neutralino mass matrix reads ($c_\beta \equiv \cos \beta$, $s_\beta \equiv \sin \beta$, $c_W \equiv \cos \theta_W$, $s_W \equiv \sin \theta_W$)

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix}, \quad (92)$$

and is diagonalized by the unitary transformation

$$N^* \mathcal{M}_N N^\dagger = \text{diag} (m_{\tilde{\chi}_1^0} \ m_{\tilde{\chi}_2^0} \ m_{\tilde{\chi}_3^0} \ m_{\tilde{\chi}_4^0}). \quad (93)$$

Summarizing, the masses and couplings of the two charginos and of the four neutralinos are characterized by four parameters: the gaugino masses M_2 and M_1 (which will be related in the following), the superpotential Higgs mass μ and $\tan \beta$. It should be noted that the lightest neutralino mass eigenstate, $\tilde{\chi}_1^0$, is the favourite candidate for being the Lightest Supersymmetric Particle (LSP) in the MSSM spectrum. An alternative candidate is the sneutrino $\tilde{\nu}_\tau$, but it is actually the LSP of the MSSM for a much smaller range of parameter space. In general, the lightest neutralino turns out to be a mixture of the four interaction eigenstates

$$\tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W}_3 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0 \quad (94)$$

The case of a pure photino, $\tilde{\chi}_1^0 = \tilde{\gamma}$, which was assumed for simplicity in some old phenomenological analyses, would correspond to the special combination $(N_{11}, N_{12}, N_{13}, N_{14}) = (\sin \theta_W, \cos \theta_W, 0, 0)$, but there is no theoretical reason to prefer it.

5.2.2 Non-minimal alternatives to the MSSM

The assumptions defining the MSSM are plausible but not compulsory. Relaxing them leads to non-minimal supersymmetric extensions of the SM, which typically increase the number of free parameters without (at present) a corresponding increase of physical motivation. We mention here two popular options.

The simplest non-minimal model [42] is constructed by adding to the MSSM a gauge-singlet Higgs superfield N , and by requiring purely trilinear superpotential couplings. Folklore arguments in favour of this model are that it avoids an explicit supersymmetric mass parameter $\mu \sim G_F^{-1/2}$, and that the homogeneity properties of its superpotential recall the structure of the simplest superstring effective theories. These statements, however, are not based on solid theoretical ground, and counterarguments exist.

In the formulation of the MSSM, the assumption of exact R -parity is of crucial importance, since relaxing it can drastically modify the phenomenological signatures. In fact, by imposing discrete symmetries weaker than R -parity we can allow for some of the terms in the last two lines of eq. (59), and therefore for explicit R -parity breaking, in a phenomenologically acceptable way [43] (for a recent review on the phenomenology of explicit R -parity breaking, see e.g. [44]). Another possibility [45] is that R -parity is spontaneously broken by the VEV of a sneutrino field, but it is by now experimentally ruled out by LEP data if we stick to the MSSM field content.

5.2.3 The MSSM RGEs and radiative $SU(2) \times U(1)$ breaking

We shall now assume, for the rest of this section, that the MSSM can be safely extrapolated up to a very large scale M . We shall also assume that, at the very large scale M , we can assign universal boundary conditions on the soft terms, in the form of a universal scalar mass (m_0^2), a universal gaugino mass ($m_{1/2}$), and a universal cubic scalar coupling (A_0), all of the order of the electroweak scale. Then the values of the MSSM parameters at the electroweak scale are strongly correlated by the corresponding RGEs, whose main features and implications will be now discussed.

We begin by spelling out in more detail the assumptions on the boundary conditions. For definiteness, we identify here the scale M with the supersymmetric grand-unification scale $M_U \sim 2 \times 10^{16}$ GeV, to be defined later. We then assume that, in first approximation, at the scale M the running gauge coupling constants obey the relations of eq. (45). Similarly, we assume for the gaugino masses

$$M_3(M) = M_2(M) = M_1(M) \equiv m_{1/2}, \quad (95)$$

for the soft supersymmetry breaking scalar masses

$$\begin{aligned} \tilde{m}_Q^2(M) &= \tilde{m}_{U^c}^2(M) = \tilde{m}_{D^c}^2(M) = \tilde{m}_L^2(M) \\ &= \tilde{m}_{E^c}^2(M) = m_{H_1}^2(M) = m_{H_2}^2(M) = m_0^2, \end{aligned} \quad (96)$$

and for the soft supersymmetry-breaking scalar couplings

$$A^U(M) = A^D(M) = A^E(M) = A_0. \quad (97)$$

We stress that, while (45) and (95) can be justified in models of supersymmetric grand unification, the universal structure in generation space of (96) and (97) requires a deeper justification in the underlying theory of spontaneous supersymmetry breaking. Counting also the supersymmetric Higgs mass $\mu(M) = \mu_0$ and the supersymmetry-breaking Higgs mixing term $m_3^2(M) = (m_3^2)_0$, in addition to the gauge and Yukawa couplings we have in the MSSM five more parameters

$$\mu_0, \quad m_{1/2}, \quad m_0^2, \quad A_0, \quad (m_3^2)_0, \quad (98)$$

which control the low-energy effective Lagrangian (62).

Some of the RGEs for the MSSM parameters have quite simple approximate solutions. For example, those for the gauge couplings have the form of eq. (44) and are solved as in eq. (46), the only difference being the one-loop beta function coefficients. Those appropriate to the MSSM are [46]

$$b_3 = -3, \quad b_2 = 1, \quad b_1 = \frac{33}{5}. \quad (99)$$

A more detailed phenomenological discussion of the constraints on the low-energy gauge-couplings will be given in the next subsection, after introducing the concept of supersymmetric grand unification.

For the gaugino masses, similar equations hold:

$$\frac{dM_A}{dt} = \frac{b_A}{8\pi^2} g_A^2 M_A, \quad (A = 1, 2, 3), \quad (100)$$

and they are also immediately solved with the boundary conditions (95), to give

$$M_A(Q) = \frac{g_A^2(Q)}{g_U^2} m_{1/2}. \quad (101)$$

Numerically, this corresponds to $M_3 \sim 3 m_{1/2}$, $M_2 \sim 0.85 m_{1/2}$, $M_1 \sim 0.25 m_{1/2}$, with possible corrections due to higher-loops and threshold effects.

Neglecting intergenerational mixing, the one-loop RGE for the top Yukawa coupling reads [47]

$$\frac{dh_t}{dt} = \frac{h_t}{8\pi^2} \left(-\frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{13}{18}g'^2 + 3h_t^2 + \frac{1}{2}h_b^2 \right), \quad (102)$$

with similar equations for the bottom and tau Yukawa couplings, h_b and h_τ . A close look at the above RGE, combined with the experimental knowledge of the top and bottom quark masses, can give us important informations.

Consider first the simple case of $\tan \beta \ll M_t/M_b$. In first approximation, we can neglect the effects of the (g, g') gauge couplings and of the (h_b, h_τ) Yukawa couplings on the running of the top Yukawa coupling, h_t . Then we can immediately realize that the RGE for the top Yukawa coupling, eq. (102), admits an effective infrared fixed point [48], smaller than in the SM case [49]. Whatever high value one assigns to the top Yukawa coupling at the large scale M , the top Yukawa coupling at the electroweak scale never exceeds a certain maximum value, $\alpha_t^{max} \simeq (8/9)\alpha_S$, where $\alpha_t \equiv h_t^2/(4\pi)$ and $\alpha_S \equiv g_3^2/(4\pi)$. Remembering the tree-level formula for M_t , this suggests the lower bound

$$\tan \beta \gtrsim 2. \quad (103)$$

However, a precise bound can be established only after the inclusion of the possibly sizeable radiative corrections associated with threshold effects, both at the unification scale and at the electroweak scale [50], combined with two-loop RGEs. As a result, values of $\tan \beta$ as low as 1.6 may still be acceptable. The bounds of course evaporate if we allow for the possible existence of new physics thresholds between the electroweak and the grand-unification scales.

This infrared structure becomes even more interesting if we include the effects of the bottom-quark Yukawa coupling, so that also large values of $\tan \beta$ can be considered. In this case, the top and bottom Yukawa couplings admit an effective infrared fixed curve, approximately described by [51]

$$\alpha_t + \alpha_b \lesssim \frac{8}{9}\alpha_S f(\alpha_t, \alpha_b), \quad (104)$$

where f is a hypergeometric function bounded by $1 \leq f \leq 12/7$. This translates into the approximate bound

$$\frac{M_t^2}{\sin^2 \beta} + \frac{M_b^2}{\cos^2 \beta} \lesssim (200 \text{ GeV})^2. \quad (105)$$

It is remarkable that, for a large range of $\tan \beta$ values between 1 and M_t/M_b , this bound is respected but almost saturated: several theoretical papers have been written to suggest possible explanations of this empirical observation, but such a discussion is beyond the aim of the present lectures.

Similar equations can be derived [52] for the soft supersymmetry-breaking scalar masses, for the remaining soft supersymmetry-breaking parameters $(A_t, A_b, A_\tau, m_3^2)$ and for the superpotential Higgs mass μ . Also, the inclusion of the complete set of Yukawa couplings, including mixing, is straightforward. In general, the RGE for superpotential couplings and soft supersymmetry-breaking parameters have to be solved by numerical methods (or approximate analytical methods). Exact solutions of the one-loop RGEs can be found for the squark and slepton masses of the first two generations, for which the Yukawa couplings are negligible. For example, we get $m_Q^2, m_{U^c}^2, m_{D^c}^2 \sim m_0^2 + (5 \div 8)m_{1/2}^2$,

$m_L^2 \sim m_0^2 + 0.5 m_{1/2}^2$, $m_{E^c}^2 \sim m_0^2 + 0.15 m_{1/2}^2$, with the warning that higher loops and threshold effects should be included for more accurate predictions.

One of the most attractive features of the MSSM is the possibility of describing the spontaneous breaking of the electroweak gauge symmetry as an effect of radiative corrections [53]. Notice that, starting from universal boundary conditions at the scale M_U , it is possible to explain naturally why fields carrying colour or electric charge do not acquire non-vanishing VEVs, whereas the neutral components of the Higgs doublets do. Also, the electroweak scale gets linked with the scale of the soft supersymmetry-breaking masses in the MSSM (which remains however an independent input parameter), and is stable with respect to quantum corrections.

We give here a simplified description of the mechanism, in which the physical content is transparent, and we comment later on the importance of a more refined treatment. The starting point are the boundary conditions on the model parameters at the scale M , summarized by:

$$g_U, \quad (h_t, h_b, h_\tau)_0, \quad \mu_0, \quad m_{1/2}, m_0, A_0, (m_3^2)_0. \quad (106)$$

After evolving all the running parameters from the grand-unification scale M to a low scale $Q \sim m_Z$, according to the RGEs described in the previous section, we can consider the RG-improved tree-level potential $V_0(Q)$, which has the functional form of eq.(63), but is expressed in terms of running masses and coupling constants evaluated at the scale Q . $V_0(Q)$ will describe an acceptable breaking of $SU(2) \times U(1)$ if the conditions of eqs.(66) and (67) are satisfied, together with a certain number of conditions for the absence of charge and colour breaking minima (for recent discussions, see e.g. [54]), and finally if $v^2 \equiv v_1^2 + v_2^2$ is of the right magnitude to fit the observed values of the W and Z masses, according to eq. (70). In other words, the measured values of the weak boson masses set a constraint on the independent parameters of eq. (106).

A crucial rôle in the whole process is played by the top quark mass, since the top quark Yukawa couplings governs the renormalization group evolution of the mass parameter $m_{H_2}^2$:

$$\frac{dm_{H_2}^2}{dt} = \frac{1}{8\pi^2} \left(-3g_2^2 M_2^2 - g'^2 M_1^2 + 3h_t^2 F_t \right), \quad (107)$$

where

$$F_t = m_Q^2 + m_{U^c}^2 + m_{H_2}^2 + A_t^2. \quad (108)$$

For a given set of boundary conditions on the remaining parameters, too small values of h_t are not able to drive $\mathcal{B} < 0$ at scales $Q \sim m_Z$, so that the origin remains a minimum and we end up with unbroken $SU(2) \times U(1)$; on the other hand, too large values of h_t can either drive $\mathcal{S} < 0$, which would correspond to a potential $V_0(Q)$ unbounded from below, or violate one of the conditions for the absence of charge or colour breaking minima.

The use of the renormalization group improved tree level potential, $V_0(Q)$, is very practical, but it relies on the assumption that, once all large logarithms have been included in the running parameters, all the remaining one loop corrections to the scalar potential can be neglected at the scale $Q \sim m_Z$. We know in fact that the complete expression of the one-loop effective potential is given by

$$V_1(Q) = V_0(Q) + \Delta V_1(Q), \quad (109)$$

where, neglecting a field-independent part which is proportional to $Str \mathcal{M}^2$ and contributes only to the vacuum energy,

$$\Delta V_1(Q) = \frac{1}{64\pi^2} Str \left\{ \mathcal{M}^4(Q) \left[\log \frac{\mathcal{M}^2(Q)}{Q^2} - \frac{3}{2} \right] \right\}. \quad (110)$$

Indeed, it was shown in [55] that, in order to obtain reliable results, stable under small changes of the renormalization scale Q , it is essential to use at least the full one-loop effective potential, especially if

the supersymmetry-breaking mass splittings start to be sizeable with respect to m_Z . A reasonable first approximation consists in using $V_0(Q)$, but choosing a scale \hat{Q} of the order of some average stop mass: this minimizes the threshold corrections due to the presence of many slightly different mass scales close to the electroweak scale.

To conclude the discussion of radiative symmetry breaking, we show now that in the MSSM (with universal boundary conditions) we expect

$$1 < \tan \beta < \frac{M_t}{M_b}. \quad (111)$$

The proof relies on the relation, derived from the minimization of $V_0(Q)$:

$$\left(\frac{v_2}{v_1}\right) = \frac{m_1^2 + m_Z^2/2}{m_2^2 + m_Z^2/2}. \quad (112)$$

The boundary conditions at the unification scale are $m_1^2(M) = m_2^2(M) = m_0^2 + \mu_0^2$, and the RGE for the difference $m_1^2 - m_2^2$ reads

$$\frac{d(m_1^2 - m_2^2)}{dt} = \frac{1}{8\pi^2} (3h_b^2 F_b + h_\tau^2 F_\tau - 3h_t^2 F_t), \quad (113)$$

where F_b and F_τ are (positive) quantities analogous to F_t . Imagine now that $\tan \beta < 1$, and remember the tree-level expressions for the top and bottom masses. The fact that $M_t \gg M_b$ then implies $h_t \gg h_b$, this in turn implies that at the scale \hat{Q} , where the use of $V_0(Q)$ is appropriate, $m_1^2 > m_2^2$. But eq.(112) then tells us that $\tan \beta > 1$, in contradiction with the starting assumption. Similarly we can prove that $\tan \beta < M_t/M_b$.

As a final remark, we stress a problem left unsolved by the MSSM description of radiative symmetry breaking: the scale of the soft terms, which in turn determines the electroweak scale, is not dynamically determined, but introduced ‘by hand’ in the boundary conditions on the mass parameters. To discuss the possible dynamical determination of such a scale, needed for a fully satisfactory solution of the naturalness problem, we need a theory of spontaneous supersymmetry breaking.

5.3 Supersymmetric Grand Unification

Some of the problems of non-supersymmetric unification, including those with proton decay and with the low-energy values of the gauge coupling constants, may find a natural solution with the incorporation of supersymmetry. The minimal model of supersymmetric grand unification [56] is based on $SU(5)$, and is constructed in analogy with the MSSM. Gauge bosons and matter fermions fall in the same $SU(5)$ representations as in the Georgi-Glashow model, but are promoted to the corresponding supermultiplets. The Higgs sector is extended to the following chiral superfields: $H(5)$, $\overline{H}(\overline{5})$ and $\Sigma(24)$. The VEV of the adjoint scalar, $\langle \Sigma \rangle = V \cdot \text{diag}(2, 2, 2, -3, -3)$ breaks $SU(5)$ down to the SM gauge group, whereas $\langle H \rangle = (0, 0, 0, 0, v_2)$ and $\langle \overline{H} \rangle = (0, 0, 0, 0, v_1)$ describe the breaking of the electroweak symmetry. The superpotential is of the form

$$w = h \cdot 10_F \times 10_F \times H + h' \cdot 10_F \times \overline{5}_F \times \overline{H} + M' H \overline{H} + \lambda_1 H \Sigma \overline{H} + M \text{Tr} \Sigma^2 + \lambda_2 \text{Tr} \Sigma^3. \quad (114)$$

The breaking of $SU(5)$ must preserve supersymmetry and give mass to the color triplet Higgs bosons, while keeping their doublet partners light. Looking at the equations of motion for the auxiliary fields, we find that $V \sim M/\lambda_2$ and, in order to keep the Higgs doublets light, $M' \simeq 3\lambda_1 V$. The fine-tuning related to this last condition is at the origin of the so-called doublet-triplet splitting problem of minimal supersymmetric grand unification. The superheavy vector bosons have masses proportional to $g_U V$, the Higgs triplets in the fundamental and anti-fundamental have masses proportional to $\lambda_1 V$, and the Higgs particles in the adjoint have masses proportional to $\lambda_2 V$. After decoupling these heavy states, and

introducing by hand some soft supersymmetry-breaking mass terms, we are left with the MSSM as the effective theory at scales $Q \ll M_U$.

In the leading logarithmic approximation, the predictions of supersymmetric grand-unification just depend on the MSSM particle content. Assuming for simplicity that all supersymmetric particles have masses of order m_Z , we obtain [46] $M_U \simeq 2 \times 10^{16}$ GeV (which increases the proton lifetime for gauge-boson-mediated processes beyond the present experimental limits) and $\sin^2 \theta_W \simeq 0.23$. At the time of [46], when data were pointing towards a significantly smaller value of $\sin^2 \theta_W$, this was considered by some a potential phenomenological shortcoming of the MSSM. The high degree of compatibility between data and supersymmetric grand unification became manifest [28] only later, after improved data on neutrino-nucleon deep inelastic scattering were obtained, and was progressively reinforced by the subsequent LEP and Tevatron data. We should not forget, however, that unification of the MSSM is not the only solution which can fit the values of the gauge coupling constants at the electroweak scale extracted from experiment: for example, non-supersymmetric models with *ad hoc* light exotic particles or intermediate symmetry-breaking scales could also do the job. The MSSM, however, stands out as the simplest physically motivated solution.

In models of supersymmetric grand-unification, including the minimal one, we still find the conventional mechanisms for proton decay, described by supersymmetric $d = 6$ operators. Gauge-boson exchange, however, does not lead to proton decay at a detectable rate, since the unification mass M_U is more than one order of magnitude higher than in the non-supersymmetric case, and the proton lifetime scales as M_U^4 . Color-triplet Higgs boson exchange could lead to decay modes such as $p \rightarrow \mu^+ K^0$ or $\bar{\nu}_\mu K^+$, but the corresponding rate would be undetectably small, being proportional to some Yukawa coupling squared, if the triplet masses are of the order of M_U . However, as pointed out in [57], supersymmetric models admit a new class of $d = 5$ operators which, when dressed by loops of MSSM particles, may lead to a proton lifetime proportional to $\Delta m^2 M_U^2$ instead of M_U^4 , with distinctive decay modes such as $p \rightarrow K^+ \bar{\nu}_\mu$. This is indeed the case of minimal supersymmetric $SU(5)$. However, the detailed predictions for the decay rates are rather model-dependent, since they are controlled by superpotential couplings containing two arbitrary phases and three independent superheavy masses, and by the details of the MSSM particle spectrum.

If we want to make the comparison between low-energy data and the predictions of specific grand-unified models more precise, there are several factors that should be further taken into account. After the inclusion of higher-loop corrections and threshold effects, eq. (46) is modified as follows

$$\frac{1}{g_A^2(Q)} = \frac{1}{g_U^2} + \frac{b_A}{8\pi^2} \log \frac{M_U}{Q} + \Delta_A^{th} + \Delta_A^{l>1} \quad (A = 1, 2, 3). \quad (115)$$

In eq. (115), Δ_A^{th} represents the so-called *threshold effects*, which arise whenever the RGE are integrated across a particle threshold [58], and $\Delta_A^{l>1}$ represents the corrections due to two- and higher-loop contributions to the RGE [59]. Both Δ_A^{th} and $\Delta_A^{l>1}$ are scheme-dependent, so one should be careful to compare data and predictions within the same renormalization scheme. Δ_A^{th} receives contributions both from thresholds around the electroweak scale (top quark, Higgs boson, and in SUSY-GUTs also the additional particles of the MSSM spectrum), and from thresholds around the grand-unification scale (superheavy gauge and Higgs bosons, and in SUSY-GUTs also their superpartners). Needless to say, these last threshold effects can be computed only in the framework of a specific grand-unified model, and typically depend on a number of free parameters. Besides the effects of gauge couplings, $\Delta_A^{l>1}$ must include also the effects of Yukawa couplings, since, even in the simplest mass-independent renormalization schemes, gauge and Yukawa couplings mix beyond the one-loop order. In minimal $SU(5)$ grand unification, and for sensible values of the top and Higgs masses, all these corrections are small and do not affect substantially the conclusions derived from the naïve one-loop analysis. This is no longer the case, however, for supersymmetric grand unification. First of all, one should notice that the MSSM by itself does not uniquely define a SUSY-GUT, whereas threshold effects and even the proton lifetime (due to the

new class of diagrams [57] which can be originated in SUSY-GUTs) become strongly model-dependent. Furthermore, the simplest SUSY-GUT [56], containing only chiral Higgs superfields in the 24, 5 and $\bar{5}$ representations of $SU(5)$, has a severe problem in accounting for the huge mass splitting between the $SU(2)$ doublets and the $SU(3)$ triplets sitting together in the 5 and $\bar{5}$ Higgs supermultiplets. Threshold effects are typically larger than in ordinary GUTs, because of the much larger number of particles in the spectrum, and in any given model they depend on several unknown parameters. Also two-loop effects of Yukawa couplings are quantitatively important in SUSY-GUTs, since they depend not only on the heavy quark masses, but also on $\tan \beta$: these effects are maximal for $\tan \beta$ close to 1 or to M_t/M_b , which correspond to a strongly interacting top or bottom Yukawa coupling. There is no problem of principle in evaluating all these effects, but they introduce a large amount of model-dependence when we try to push the comparison between theory and experiment to the level of the present experimental precision. The conclusion is that, even imagining a further reduction in the errors of the experimental determinations of the low-energy gauge couplings, it is impossible to claim indirect evidence for supersymmetry and to predict the MSSM spectrum with any significant accuracy. The only safe statement is that, at the level of precision corresponding to the naïve one-loop approximation, there is a remarkable consistency between experimental data and the prediction of supersymmetric grand unification, with the MSSM R -odd particles roughly at the electroweak scale.

To conclude the discussion of supersymmetric grand unification, it is worth spending a few words on how its phenomenologically successful prediction of the low-energy gauge couplings could be embedded within our candidate theories of all interactions, namely superstring theories or, according to the most recent developments, the M-theory underlying all superstring theories.

Traditionally, the discussion of the unification of all couplings used to be given in the context of the perturbative formulation of four-dimensional heterotic string models. In such a context, the only free parameter is the string tension, which fixes the unit of measure of the massive string excitations. All the other scales and parameters are related to VEVs of scalar fields, the so-called *moduli*, corresponding to flat directions of the scalar potential. In particular, there is a relation among the string mass $M_S \sim \alpha'^{-1/2}$, the Planck mass $M_P \sim G_N^{-1/2}$, and the unified string coupling constant g_{string} , which reflects unification with gravity, and implies that in any string vacuum one has (at least in principle) one more prediction than in ordinary field-theoretical grand unification. In a large class of perturbative string models, we can write down an equation of the same form as (115), and compute g_U , M_U , Δ_A^{th} , ... in terms of the relevant VEVs [60]. So doing, we find $M_U \simeq 0.7 \times g_U \times 10^{18}$ GeV, more than one order of magnitude higher than the naïve MSSM extrapolations from low-energy data. This is the so-called string unification problem. Several suggestions for its solution have been put forward: an intermediate phase of conventional field-theoretical unification between M_U and M_S , large string threshold corrections, intermediate scales, etc.. An intriguing observation was made recently in connection with the newly discovered non-perturbative string dualities. In the strong coupling limit, the $E_8 \times E_8$ heterotic string leads to a new dimension which is slightly different from the familiar ten dimensions that are usually considered in the perturbative discussion of heterotic string compactifications. Instead of being similar to a circle, it is more like a segment [61]. The gauge fields and matter live at the endpoints only, while gravity propagates in the bulk. Suppose that a fifth dimension of this type exists below the unification scale. Since the MSSM fields live in the walls, the evolution of the gauge couplings is the standard four-dimensional one. Since gravity propagates in the full five dimensions, however, the effective gravitational coupling runs faster than in four dimensions. For a fifth dimension of the appropriate size, the kink in the gravitational coupling can make all couplings meet [62] at the unification scale M_U . Of course, this is not more predictive than ordinary grand unification, since the size of the fifth dimension can be taken as a parameter, but it shows that the string unification problem may be solved in some appealing way.

5.4 Supersymmetry breaking

An important criterion for supersymmetry breaking follows directly from the basic anticommutation relation of the supersymmetry algebra, eq. (54), remembering that $H \equiv P_0$ is the Hamiltonian: . If the Hilbert space has positive norm, supersymmetry is spontaneously broken if and only if the Hamiltonian does not annihilate the vacuum, $H|0\rangle \neq 0$. This corresponds in turn to having a positive vacuum energy, $\langle V \rangle > 0$. Remembering the structure of the scalar potential in renormalizable theories with global supersymmetry, eq. (56), the condition for supersymmetry breaking is then that at least one of the auxiliary fields of the chiral and vector supermultiplets has a non-vanishing VEV,

$$\langle F_i \rangle \neq 0 \quad \text{and/or} \quad \langle D^a \rangle \neq 0. \quad (116)$$

The unavoidable consequences of the spontaneous breaking of global supersymmetry are then

- The existence of a massless fermion, the *goldstino*, residing in the superfields whose auxiliary fields acquire non-vanishing VEVs (in complete analogy with the goldstone bosons of ordinary spontaneously broken continuous global symmetries).
- A positive vacuum energy (we shall describe in a moment what happens when the coupling to supergravity is introduced).
- Some phenomenologically unacceptable mass relations, such as $\text{Str } \mathcal{M}^2 = 0$ in each separate sector of the spectrum. It should be kept in mind, however, that such a relation is valid only at the classical level, and in the absence of non-renormalizable interactions and anomalous $U(1)$ factors.

The general, ‘kinematical’ aspects of spontaneous supersymmetry breaking are well understood, both in the global [63] and in the local [64] case: in a $N = 1$, $d = 4$ theory with chiral and vector supermultiplets, the order parameters controlling supersymmetry breaking are the VEVs of the associated auxiliary fields, F^i and D^a , which give a positive semi-definite contribution to the scalar potential. For supersymmetry breaking to be compatible with a flat space-time background, the inclusion of gravitational interactions is essential, since in Poincaré supergravity the scalar potential reads [65]

$$V = ||F||^2 + ||D||^2 - ||H||^2. \quad (117)$$

The three terms $||F||^2$, $||D||^2$ and $||H||^2$ are positive-semidefinite, and controlled by the auxiliary fields of the chiral, vector and gravitational supermultiplets, respectively. The first two terms have different expressions but identical rôles in local and global supersymmetry; the third one, peculiar to supergravity, has the universal property that $\langle ||H||^2 \rangle = 3 m_{3/2}^2 M_P^2$, where $m_{3/2}$ is the mass of the spin-3/2 gravitino (the supersymmetric partner of the spin-2 graviton) and $M_P \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the Planck mass.

As will be clear in a moment, to generate phenomenologically acceptable masses for the supersymmetric partners of ordinary particles, a realistic model must have

$$\Lambda_S \equiv \langle ||F||^2 + ||D||^2 \rangle^{1/4} \gtrsim G_F^{-1/2}. \quad (118)$$

On the other hand, to satisfy the present bounds on the cosmological constant, a realistic model must also have

$$\Lambda_{cosm} \equiv \langle V \rangle^{1/4} \lesssim 10^{-4} \text{ eV} \sim G_F^{-1} M_P^{-1}. \quad (119)$$

It is then obvious that, when discussing the vacuum energy, the gravitational contribution to the scalar potential must be essentially identical to the non-gravitational one. However, as we shall see in the following, there are situations in which gravitational interactions can be neglected when restricting the attention to the spectrum and the interactions relevant for present accelerator experiments.

The goldstino \tilde{G} , which provides the $\pm 1/2$ helicity components of the massive gravitino via the super-Higgs mechanism, is determined by

$$\tilde{G} = \langle F_i \rangle \psi^i + \langle D_a \rangle \lambda^a. \quad (120)$$

The mass splittings in the different sectors of the model, denoted here schematically with a sub-index I , are controlled by

$$(\Delta m^2)_I \sim \lambda_I \Lambda_S^2, \quad (121)$$

where λ_I is the effective coupling of the goldstino supermultiplet to the sector I . This is true not only at tree level, but also after the inclusion of quantum corrections, since the latter can be incorporated in a local effective Lagrangian, which must exhibit the spontaneous nature of supersymmetry breaking if a full, non-anomalous set of supersymmetric multiplets is kept. In order for supersymmetry to solve the naturalness problem, it is customary to require that the mass splittings among the MSSM states be $(\Delta m^2)_I \sim G_F^{-1}$. However, this is not sufficient to fix Λ_S or, equivalently, $m_{3/2}$ (to an excellent approximation, $\Lambda_S = \sqrt{3 m_{3/2} M_P}$): according to the numerical values of the effective couplings λ_I , different possibilities arise, to be described in the following paragraphs.

Despite the satisfactory understanding of the ‘kinematical’ aspects of spontaneous supersymmetry breaking, what we are still lacking is some compelling idea about the symmetries and dynamics that control such a phenomenon in the fundamental theory of Nature, and explain the origin of the different scales relevant for the problem: Δm^2 , Λ_S and Λ_{cosm} . This is a very difficult and ambitious problem, and it is not surprising that a final solution has not been found yet. Several interesting ideas have been pursued in recent years, but there are still many open problems. We just mention here some of the existing approaches, referring the reader to the literature for more details. For a recent review of the possible mechanisms of supersymmetry breaking, see e.g. [66]. One interesting possibility is that, in the context of supergravity, the spontaneous breaking of supersymmetry finds its origin in non-perturbative phenomena, such as gaugino condensation [67]. Explicit models of this type exist, but they have to rely on some *ad hoc* assumptions: being supergravity an effective, non-renormalizable theory, it is difficult to control quantum corrections already at the perturbative level. Another possibility is spontaneous breaking at the string level, via coordinate-dependent compactifications [68]. There are however unsolved problems such as the mechanism for the stabilization of the dilaton VEV and the generic instability of string vacua with broken supersymmetry and vanishing cosmological constant with respect to string loop corrections. The present hope is that some more insight into this mechanism, which may lead to a non-perturbative formulation of it, could be gained by exploiting the recently discovered string dualities. A different approach to the study of spontaneous supersymmetry breaking consists in working at the level of renormalizable gauge theories with global supersymmetry, and in posing dynamical questions of more limited scope. Despite the encouraging results in recent years (for reviews, see e.g. [69]), models of dynamical supersymmetry breaking at low energy are still quite contrived when one tries to make them realistic.

Given this state of affairs, in the following we shall give a macroscopic description of the different scenarios for spontaneous supersymmetry breaking, trying to emphasize their generic features and phenomenological implications, and avoiding the discussion of the details of the microscopic theory.

5.4.1 Supergravity models with heavy gravitino

The first possibility, realized in the so-called hidden-sector supergravity models, is that the couplings of the goldstino supermultiplet to the MSSM states are of gravitational strength, $\lambda_I \sim \Lambda_S^2/M_P^2$. In this case the desired MSSM spectrum requires $\Lambda_S \sim G_F^{-1/4} M_P^{1/2} \sim 10^{10} \div 10^{11}$ GeV, and therefore $m_{3/2} \sim G_F^{-1/2}$. The effective theory at the electroweak scale is obtained from the underlying supergravity by taking formally the limit $M_P \rightarrow \infty$, while keeping $m_{3/2}$ fixed [70]: this gives precisely the MSSM with explicitly but softly broken supersymmetry. The states with masses $\mathcal{O}(m_{3/2})$ and interactions of gravitational strength need not be included in the effective theory.

In the minimal realization of such a scenario, the superfield content of the model can be classified in two distinct sectors: the ‘observable’ sector, containing the MSSM states, and the ‘hidden’ sector, containing at least the gravitational supermultiplet and the goldstino supermultiplet (for definiteness, we assume here that it is a gauge singlet chiral superfield, S). The two sectors are connected only via non-renormalizable interactions, suppressed by inverse powers of the Planck mass. The scale of supersymmetry breaking is given by $\langle F_S \rangle \sim G_F^{-1/2} M_P$, and the fermionic component of S is the goldstino \tilde{G} . The gravitino mass is $m_{3/2} \sim \langle F_S \rangle / M_P \sim G_F^{-1/2}$, and the SUSY-breaking mass splittings, both in the observable and in the hidden sector, are of the order of the gravitino mass, since they are originated by tree-level couplings of gravitational strength. In contrast with the case of renormalizable, global supersymmetry, the supertrace mass sum rule is in general violated, and the mass scale characterizing such violation is the gravitino mass.

Hidden-sector supergravity models exhibit some generic problems that should be solved by a satisfactory mechanism for spontaneous supersymmetry breaking, and can be summarized as follows:

- **Classical vacuum energy.** The potential of $N = 1$ supergravity does not have a definite sign and scales as $m_{3/2}^2 M_P^2$: already at the classical level, we must arrange for the vacuum energy to be vanishingly small with respect to its natural scale.
- **$(m_{3/2}/M_P)$ hierarchy.** In a theory where the only explicit mass scale is the reference scale M_P (or the string scale), we must find a convincing explanation of why it is $m_{3/2} \lesssim 10^{-15} M_P$ (as required by a natural solution to the hierarchy problem), and not $m_{3/2} \sim M_P$.
- **Stability of the classical vacuum.** Even assuming that a classical vacuum with the above properties can be arranged, the leading quantum corrections to the effective potential of $N = 1$ supergravity scale again as $m_{3/2}^2 M_P^2$, too severe a destabilization of the classical vacuum to allow for a predictive low-energy effective theory.
- **Universality of squark/slepton mass terms.** As will be discussed later, such a condition (or alternative but equally stringent ones) is phenomenologically necessary to adequately suppress FCNC, but is not guaranteed in the presence of general field-dependent kinetic terms.

From the above list, it should already be clear that the generic properties of $N = 1$ supergravity are not sufficient for a satisfactory supersymmetry-breaking mechanism. Indeed, no fully satisfactory mechanism exists, but interesting possibilities arise within string effective supergravities. The best results obtained so far are listed below:

- It is possible to formulate supergravity models where the classical potential is manifestly positive-semidefinite, with a continuum of minima corresponding to broken supersymmetry and vanishing vacuum energy, and the gravitino mass sliding along a flat direction [71, 72].
- This special class of supergravity models emerges naturally, as a plausible low-energy approximation, from four-dimensional string models, irrespectively of the specific dynamical mechanism that triggers supersymmetry breaking. Due to the special geometrical properties of string effective supergravities, the coefficient of the one-loop quadratic divergences in the effective theory, $\text{Str } \mathcal{M}^2$, can be written as [73]

$$\text{Str } \mathcal{M}^2(z, \bar{z}) = 2 Q m_{3/2}^2(z, \bar{z}), \quad (122)$$

where Q is a field-independent coefficient, calculable from the modular weights of the different fields belonging to the effective low-energy theory, i.e. the integer numbers specifying their transformation properties under the relevant duality. The non-trivial result is that the only field-dependence of $\text{Str } \mathcal{M}^2$ occurs via the gravitino mass. Since all supersymmetry-breaking mass splittings, including those of the massive string states not contained in the effective theory, are proportional to the gravitino mass, this sets the stage for a natural cancellation of the $\mathcal{O}(m_{3/2}^2 M_P^2)$

one-loop contributions to the vacuum energy. Indeed, there are explicit string examples that exhibit this feature. If this property can persist at higher loops (an assumption so far), then the hierarchy $m_{3/2} \ll M_P$ can be induced by the logarithmic corrections due to light-particle loops [72].

- In this special class of supergravity models one naturally obtains, in the low-energy limit where only renormalizable interactions are kept, very simple mass terms for the MSSM states, calculable via simple algebraic formulae from the modular weights of the corresponding fields and easily reconcilable with the phenomenological universality requirements [73]. This last result can indeed be obtained also in a slightly less restrictive framework [74].

5.4.2 Supergravity models with light gravitino

The second possibility occurs when the goldstino supermultiplet is coupled to the MSSM sector by gauge or Yukawa interactions, much stronger than the gravitational interactions. Taking for example $\lambda_I \sim 1$, to get the desired mass splittings one needs $\Lambda_S \sim G_F^{-1/2}$, giving $m_{3/2} \sim G_F^{-1} M_P^{-1} \sim (\text{few}) 10^{-5} \text{ eV}$. If there is some weak coupling and the goldstino supermultiplet couples to the MSSM states only via loops, Λ_S and $m_{3/2}$ can increase by a few orders of magnitude, since the effective couplings λ_I can be suppressed by numerical factors such as $\alpha/(4\pi)$ and by mass ratios such as Λ_S/M , where $M \geq \Lambda_S$ is some supersymmetry-preserving mass term, possibly associated with the vacuum expectation value of a standard-model-singlet scalar field. In this second class of models, gravitational interactions are relevant only for the discussion of the vacuum energy, and the effective theory at the electroweak scale can be obtained by taking formally the naïve limit $M_P \rightarrow \infty$, while keeping Λ_S constant [75].

A low scale of supersymmetry breaking, Λ_S , may be favoured by generic arguments related with the flavour problem. In the MSSM, the most general set of soft supersymmetry-breaking terms introduces many new sources of flavour violation, besides the Yukawa couplings in the superpotential: as will be discussed later, only non-generic choices of the soft terms (approximate universality or alignment) can lead to an acceptable phenomenology. From the point of view of the underlying theory with spontaneous supersymmetry breaking, the typical magnitude of the soft terms in the sfermion sector is Λ_S^2/Λ , where Λ is the scale suppressing the corresponding nonrenormalizable operators in the Kähler potential. If the scale of flavour physics, Λ_{flav} , is larger than Λ , then we would expect flavour-breaking effects on the soft terms to be suppressed by Λ/Λ_{flav} , and a phenomenologically acceptable pattern of soft mass terms could naturally arise. The opposite situation, $\Lambda_{flav} \lesssim \Lambda$, would generically induce unsuppressed flavour violations in the soft terms. These generic arguments are not conclusive, but may be taken as an additional motivation to study models where Λ_S and Λ are as low as possible.

A presently popular realization of the light gravitino case is given by the so-called ‘messenger’ or ‘gauge-mediated’ models (for a recent review and references, see e.g. [76]). In the minimal version of such models, the field content can be divided into three sectors: an ‘observable’ sector, containing the MSSM fields; a ‘messenger’ sector, containing real representations of a grand-unified gauge group (for example, a $5 + \bar{5}$ of $SU(5)$, to be denoted by M and \bar{M} , respectively), which interacts with observable sector only via SM gauge interactions; a ‘secluded’ sector, containing at least the gravitational supermultiplet and the goldstino supermultiplet S , which has superpotential interactions with the messenger sector, but is decoupled at tree-level from the observable sector. If supersymmetry is spontaneously broken on the vacuum, one expects that the spectrum in the messenger sector is controlled by the combination of supersymmetric mass terms, proportional to $\langle S \rangle$, and supersymmetry-breaking masses, proportional to $\sqrt{\langle F_S \rangle}$. In the observable sector, supersymmetry breaking masses are generated by loop diagrams with messenger fields on the internal lines. For example, gaugino masses are generated at one loop, and have the form

$$M_A \sim \frac{\alpha_A}{4\pi} \frac{\sqrt{\langle F_S \rangle}}{\langle S \rangle} \cdot \sqrt{\langle F_S \rangle}, \quad (123)$$

whereas universal scalar masses are generated at two loops, and have the form

$$m_0^2 \sim \left(\frac{\alpha_A}{4\pi} \right)^2 \frac{\langle F_S \rangle}{\langle S \rangle^2} \cdot \langle F_S \rangle. \quad (124)$$

It is easy to identify in the above formulae the effective couplings of the goldstino supermultiplets to the observable sector, once the effects of loop diagrams have been included. The nice feature of these models is the fact that, due to the universal character of gauge interactions, the soft scalar masses in the observable sector are automatically universal. However, because of a Peccei-Quinn symmetry, neither μ nor $m_{\frac{2}{3}}^2$ can be generated by gauge interactions alone, so the minimal messenger model must be complicated with some superpotential interactions in order to become realistic. Once superpotential interactions are introduced, however, the universality properties of the scalar mass terms are no longer guaranteed in general. Moreover, if there is no mixing with the MSSM states, and a conserved global messenger number can be identified, then we expect a stable messenger, which may give rise to cosmological problems. Both the difficulties mentioned above can be solved by complicating sufficiently the model, but, as a result, no unique candidate messenger model is singled out.

In view of the above considerations, a more model-independent approach to the light gravitino case may be followed (for an extensive discussion, see e.g. [77]). It consists in writing down an effective theory for the light multiplets, i.e. the MSSM fields and the gravitino, assuming that the heavier fields (for example, the messengers, but not necessarily so) have been integrated out. Such an effective theory has both supersymmetry and the gauge symmetry linearly realized on the fields, but non-renormalizable operators are present to encode the low-energy effects of the underlying dynamics. In this theory, supersymmetry is spontaneously broken, and masses and couplings can be read off tree-level formulae directly. The limit of such an approach is the lower amount of predictive power, but the advantage is the possibility of an efficient parametrization of the model-independent aspects of the resulting phenomenology. In particular, the differences with the heavy gravitino case become more and more important as the supersymmetry-breaking scale Λ_S , suppressing the non-renormalizable operators, gets closer and closer to the weak scale. We finally remark that an effective theory of this kind is valid only in a limited energy range, bounded from above by unitarity, which essentially dictates, besides $\Delta m \lesssim \Lambda_S$, also $E \lesssim \Lambda_S^2/\Delta m$: new (elementary or composite) degrees of freedom must be introduced before or near this critical scale to restore unitarity.

5.5 Supersymmetric phenomenology

Let us assume, for now, exact R-parity conservation. Then:

- supersymmetric (R-odd) particles are produced in pairs: single production in reactions initiated by ordinary (R-even) particles would violate R-parity;
- supersymmetric (R-odd) particles always decay into final states involving an odd number of supersymmetric (R-odd) particles;
- the lightest supersymmetric particle (LSP) is absolutely stable.

If the LSP is neutral and weakly interacting (typical candidates encountered in model-building are the lightest neutralino or one of the sneutrinos in heavy gravitino models, and the gravitino itself in light gravitino models), then it is a possible candidate for dark matter. In collider phenomenology, being essentially invisible to the detectors, the LSP can be characterized by a distinctive missing-energy signature. Three broad scenarios for supersymmetric phenomenology then emerge, whose general features will be now described.

Heavy gravitino

This corresponds to $m_{3/2} \sim 10^2 \div 10^4$ GeV, or $\Lambda_S \sim 10^{10} \div 10^{11}$ GeV. As discussed before, in the heavy gravitino case all polarization states of the massive gravitino couple with gravitational strength, and the MSSM with soft terms is an adequate description up to energy scales of order M_U . The two most distinctive phenomenological features are that non-renormalizable operators correcting the MSSM are completely negligible at present accelerator energies, and that the LSP belongs to the MSSM spectrum.

Light gravitino

This corresponds to $m_{3/2} \sim 10^{-1} \div 10^3$ eV, or $\Lambda_S \sim 10^4 \div 10^6$ GeV. In this case, the $\pm 1/2$ helicity components of the gravitino, corresponding to the would-be goldstino, couple with strength much greater than gravitational, but still smaller than the typical strength of the gauge interactions or of the Yukawa interactions of heavy fermions. In this case, the new non-renormalizable interactions, correcting the MSSM and associated with supersymmetry breaking, are too weak to play a role in the production processes of R-odd particles, but may play an important rôle in their decays. Also, we can no longer extrapolate the MSSM up to M_U , since tree-level unitarity is violated at a critical energy $E_c \sim \Lambda_S^2/\Delta m$, and new (elementary or composite) degrees of freedom must be introduced before or near this critical scale to restore unitarity.

An important property controlling the phenomenology of these models, whose LSP is the gravitino, is the nature of the next-to-lightest supersymmetric particle (NLSP). If such particle is the lightest neutralino, for example the photino, the rate of its decay into a photon and a goldstino is given by [78]

$$\Gamma(\tilde{\gamma} \rightarrow \tilde{G}\gamma) = \frac{1}{16\pi} \frac{M_{\tilde{\gamma}}^5}{\Lambda_S^4}. \quad (125)$$

This is trivially generalized to the case of an arbitrary neutralino, as long as it has a non-negligible photino component. In this case, the typical signature of sparticle production and decay is given by photons plus missing energy. If the NLSP is a sfermion \tilde{f} , for example a stau or a sneutrino, as it may be the case in some of the messenger models, then it likes to decay into the corresponding fermion f and a goldstino. In the $m_f = 0$ limit, the decay rate reads

$$\Gamma(\tilde{f} \rightarrow \tilde{G}f) = \frac{1}{16\pi} \frac{\tilde{m}_f^5}{\Lambda_S^4}. \quad (126)$$

In this case, the phenomenology is characterized by missing energy signals, as in the standard case of heavy gravitino.

Superlight gravitino

This corresponds to $m_{3/2} \sim 10^{-6} \div 10^{-2}$ eV, or $\Lambda_S \sim 10^2 \div 10^4$ GeV. In this case, the goldstino couplings with the MSSM fields have, at the presently accessible energies, a strength comparable with the gauge couplings. As a result, it is essential to keep track, at energies of the order of the electroweak scale, of all the leading non-renormalizable interactions controlled by inverse powers of the supersymmetry-breaking scale. In fact, as we shall see in a moment, these interactions can now play an important role in the production processes: we can have not only pair-production of MSSM sparticles, but also associated production of a gravitino and a MSSM sparticle, and even pair production of gravitinos. It is also clear that in this case the effective theory has a very limited range of validity, extending not much above the electroweak scale.

To conclude the discussion of the superlight gravitino case, we would like to comment further on an intriguing aspect of its phenomenology. There may be experiments where the available energy is still insufficient for the on-shell production of other supersymmetric particles, but nevertheless sufficient

to give rise to final states with only gravitinos and ordinary particles, at measurable rates. As recently discussed in [79], powerful processes to search for a superlight gravitino \tilde{G} (when the supersymmetric partners of the Standard Model particles and of the goldstino are above threshold) are $e^+e^- \rightarrow \tilde{G}\tilde{G}\gamma$ and $q\bar{q} \rightarrow \tilde{G}\tilde{G}\gamma$, which would give rise to a distinctive (*photon + missing energy*) signal. The first process can be studied at e^+e^- colliders such as LEP or the proposed NLC, the second one at hadron colliders such as the Tevatron or the LHC. At hadron colliders, we can also consider the partonic subprocesses $q\bar{q} \rightarrow \tilde{G}\tilde{G}g$, $qg \rightarrow q\tilde{G}\tilde{G}$, $\bar{q}g \rightarrow \bar{q}\tilde{G}\tilde{G}$ and $gg \rightarrow g\tilde{G}\tilde{G}$, all contributing to the (*jet + missing energy*) signal. In the case of heavy superpartners, all these processes have cross-sections with a strong, universal power-law dependence on the centre-of-mass energy and on the scale of supersymmetry breaking, s^3/Λ_S^8 . In the absence of experimental anomalies, the above processes can be used to establish model-independent lower bounds on the gravitino mass. From the present LEP data, we can estimate $m_{3/2} \gtrsim 10^{-5}$ eV, corresponding to $\Lambda_S \gtrsim 200$ GeV. At hadron colliders, the analysis is more complicated. In the $(\gamma + \cancel{E}_T)$ channel, there are already some published D0 data collected at the Tevatron collider, from which we can extract $\Lambda_S > 245$ GeV, or $m_{3/2} > 1.4 \times 10^{-5}$ eV. We estimate that, with the presently available luminosity, the Tevatron experiments should be sensitive up to $\Lambda_S \simeq 300$ GeV, or $m_{3/2} \simeq 2.2 \times 10^{-5}$ eV. The sensitivity should be slightly higher in the (*jet + \cancel{E}_T*) channel: our estimate is $\Lambda_S \simeq 335$ GeV, or $m_{3/2} \simeq 2.7 \times 10^{-5}$ eV. At the LHC, because of the pp initial state, the most sensitive channel will be (*jet + \cancel{E}_T*), which should reach $\Lambda_S \simeq 2.2$ TeV, or $m_{3/2} \simeq 1.2 \times 10^{-3}$ eV.

As a final remark, we would like to stress that $m_{3/2} (\leftrightarrow \Lambda_S)$ is a fundamental free parameter for supersymmetric models, analogous to the Fermi constant G_F for the models of weak interactions, so it is very important to measure it or at least to bound it from below.

5.5.1 SUSY vs. electroweak precision tests

The impressive amount of data collected in recent years at LEP, at the Tevatron and elsewhere has confirmed the validity of the SM at an unprecedented level of precision. Nowadays, when discussing physics beyond the SM we must take into account that only very delicate deviations from the SM predictions are still allowed at the presently accessible energies.

In this respect, the MSSM performs very well in comparison with other candidate models. Thanks to the fact that the soft mass terms are invariant under the electroweak gauge group, the effects of virtual supersymmetric particles on observable quantities decouple in the limit of a heavy sparticle spectrum. Of course, having supersymmetric particle masses much heavier than the electroweak scale would bring back the hierarchy problem, but this is a different issue: in practice, decoupling occurs very fast and we do not need to worry about naturalness in this context. This important MSSM feature should be contrasted with examples of new physics that do not obey similar decoupling properties, such as a possible fourth fermion generation, technicolor, and others.

In the case of a heavy sparticle spectrum, the MSSM predictions for precision electroweak observables essentially coincide with those of the SM for a relatively light Higgs, and the corresponding data do not put very stringent constraints on the MSSM parameter space. In some special cases, however, a light sparticle spectrum can give rise to sizeable effects: a large stop-sbottom splitting, in the presence of relatively small soft masses for the left-handed components, can give a sizeable positive contribution to the effective ρ parameter [80]; loops involving light stops and charginos, or the top quark and the charged Higgs, may give sizeable corrections to the effective $Zb\bar{b}$ vertex, with the possibility of partial cancellations [81]; other effects related with the threshold behaviour of light charginos in the vector boson self-energies have been considered [82], but their potential impact has considerably decreased after the stringent limits on chargino masses obtained at LEP2 (see later).

In the past, given the large number of MSSM parameters, to perform global fits it was convenient to organize the data in a model-independent way, by defining a suitable approximate parametrization, and by comparing the MSSM predictions and the fits to the experimental data in terms of 3-4 relevant parameters. With the present experimental precision, this approach looks no longer adequate. In general,

the indirect bounds on the MSSM parameter space from electroweak precision data are weaker than the bounds obtained from the direct searches. Nevertheless, there are small regions of the MSSM parameter space where the indirect bounds are the most stringent ones: to discuss these bounds at the appropriate level of precision, full MSSM computations are required.

For more details on supersymmetry vs. electroweak precision data, many updated reviews are available [6, 83].

5.5.2 *SUSY vs. flavour physics*

Since the early days of supersymmetric phenomenology, it was realized [41, 84, 85] that, allowing for non-universal soft supersymmetry-breaking terms, the latter would be subject to very stringent constraints from FCNC and CP violation. An example is the decay $\mu \rightarrow e\gamma$, subject to the strong experimental bound [8] $BR(\mu \rightarrow e\gamma) < 5 \times 10^{-11}$. Off-diagonal slepton mass terms in generation space, denoted here with the generic symbol δm^2 , would contribute to the above decay at the one-loop level, via diagrams involving virtual sleptons and gauginos, and the previous limit roughly translates into $\delta m^2/m_{\tilde{l}}^2 < 10^{-3}-10^{-5}$, if one assumes gaugino masses of the order of the average slepton mass $m_{\tilde{l}}$ (a quite complicated parametrization is needed to formulate the bound more precisely). Similar constraints can be obtained by looking at the $K^0-\bar{K}^0$, $B^0-\bar{B}^0$ systems, at $b \rightarrow s\gamma$ transitions, at the electric dipole moment of the neutron, and at other flavour-changing or CP-violating phenomena. It is important to recall that all these bounds are naturally respected by the strict MSSM, where the only non-universality in the squark and slepton mass terms is the one induced by the renormalization group evolution from the cut-off scale M to the electroweak scale. However, the same bounds represent quite non-trivial requirements on extensions of the MSSM, such as supersymmetric grand-unified theories (SUSY GUTs) and string effective supergravities, since in general one expects non-universal contributions to the soft supersymmetry-breaking masses. Various mechanisms that could enforce the desired amount of universality, or, alternatively, a sufficient suppression of FCNC and CP violation without universality, have been discussed in the literature. For reviews of the theoretical and phenomenological aspects of supersymmetric flavour physics, see e.g. [86].

Moving to more general considerations, the flavour problem is one of the key issues in all extensions of the SM, including the supersymmetric ones. This is due to the fact that in the SM the $[SU(3)]^5 \times [U(1)]^4$ flavour symmetry is strongly violated, but all flavour violation is encoded in the Cabibbo-Kobayashi-Maskawa matrix, so that, thanks to the GIM mechanism, there is natural suppression of all flavour-changing and CP-violating effects. Any model of new physics must face the flavour challenge, especially if part of the new physics is close to the electroweak scale. This is certainly the case of the MSSM, where, as we have already anticipated, the supersymmetry-breaking problem and the flavour problem get mixed. Models with a light gravitino may naturally explain the absence of non-standard flavour-violating effects, whereas models with a heavy gravitino may lead to measurable signals, whose detection would open a window on the physics at very high scales.

Even ensuring that there are no tree-level FCNC, in the MSSM new contributions to FCNC processes may come from loop diagrams involving virtual non-standard particles, such as the charged Higgs boson, the stops and the charginos. Comparison with experiment may then lead to indirect constraints on the MSSM parameters. Important examples include the fits to Δm_{B_d} and $|\epsilon_K|$ and to the inclusive $b \rightarrow s\gamma$ rate. If it were possible to reduce the theoretical uncertainties due to perturbative and non-perturbative effects of the strong interactions, these processes would become a very important source of indirect limits on the MSSM spectrum.

5.5.3 *The MSSM Higgs sector*

We have seen before that, at the classical level, the MSSM is very predictive in the Higgs sector, thanks to the fact that supersymmetry forbids an arbitrary quartic term in the scalar potential. In particular, the

classical relation $m_h < m_Z$ is very constraining: if it were rigorously true, it would allow a decisive test of the MSSM already at LEP2, and today we would be very close to ruling out the MSSM! However, it is by now well known that the MSSM Higgs sector, and in particular the upper bound on the lightest Higgs boson mass, are subject to large, finite radiative corrections, dominated by loops involving the top quark and its supersymmetric partners [87]. Over the years, the original calculations were progressively refined by the inclusion of: mixing effects in the stop sector, resummation of the leading logarithms via the renormalization group, momentum dependence of the self-energies, loops of other MSSM particles, the most important two-loop corrections. The state of the art of the theoretical calculations has been recently summarized in [13, 88]. For the present value of the top quark mass, $M_t \simeq 175$ GeV, an average stop mass of 1 TeV and arbitrary stop mixing, the upper bound on m_h is approximately 125 GeV. It is perhaps worth mentioning an implicit assumption lying behind the derivation of such upper bound: non-renormalizable operators, suppressed by inverse power of Λ_S , should be negligible; indeed, one can build models with very low scales of supersymmetry breaking where this upper bound is strongly violated [77].

As a pedagogical example, we give here the explicit calculation, in a particularly simple case, of the leading radiative correction to the neutral CP -even mass matrix. Considering only the functional dependence on the fields $\varphi_i \equiv \text{Re } H_i^0$ ($i = 1, 2$), the classical potential of the MSSM can be written as

$$V_0 = m_1^2 \varphi_1^2 + m_2^2 \varphi_2^2 + 2m_3^2 \varphi_1 \varphi_2 + \frac{g^2 + g'^2}{8} (\varphi_2^2 - \varphi_1^2)^2. \quad (127)$$

The standard way of describing quantum corrections to the classical potential is to consider the effective potential, which at the one-loop level can be written as $V_1 = V_0 + \Delta V$. Including only top and stop loops, working in the \overline{DR} scheme and neglecting as usual field-independent terms, we find

$$\Delta V = \frac{3}{16\pi^2} [f(m_t^2) - f(M_t^2)], \quad f(m^2) = m^4 \left(\log \frac{m^2}{Q^2} - \frac{3}{2} \right), \quad (128)$$

where $M_t^2 = h_t^2 \varphi_2^2$ and $m_t^2 = M_t^2 + m_{\tilde{q}}^2$ are the field-dependent top and stop masses, and Q is the renormalization scale. For simplicity, we have neglected D-terms and mixing terms in the stop squark mass matrix, and we have assumed a common soft supersymmetry-breaking squark mass $m_{\tilde{q}}$.

In analogy with the tree-level case, we can use the one-loop minimization conditions,

$$\left(\frac{\partial V_1}{\partial \varphi_i} \right)_{\varphi=v} = 0, \quad (i = 1, 2), \quad (129)$$

to solve for the mass parameters m_1^2 and m_2^2 . We can then identify the one-loop-corrected entries in the neutral CP -even mass matrix with

$$(\mathcal{M}_R^2)_{ij} \equiv (\mathcal{M}_R^0)_{ij}^2 + (\Delta \mathcal{M}_R^2)_{ij} = \frac{1}{2} \left(\frac{\partial^2 V_1}{\partial \varphi_i \partial \varphi_j} \right)_{\varphi=v}. \quad (130)$$

Since in our approximation ΔV does not depend on φ_1 , we can immediately write

$$(\Delta \mathcal{M}_R^2)_{11} = (\Delta \mathcal{M}_R^2)_{12} = 0. \quad (131)$$

After some very simple algebra, we also obtain

$$(\Delta \mathcal{M}_R^2)_{22} = \frac{1}{2} \left[-\frac{1}{v_2} \left(\frac{\partial \Delta V}{\partial \varphi_2} \right)_{\varphi=v} + \left(\frac{\partial^2 \Delta V}{\partial \varphi_2^2} \right)_{\varphi=v} \right]. \quad (132)$$

From eq. (128), and the expressions for M_t^2 and m_t^2 , we get

$$\left(\frac{\partial^2 \Delta V}{\partial \varphi_2^2}\right)_{\varphi=v} = \frac{1}{v_2} \left(\frac{\partial \Delta V}{\partial \varphi_2}\right)_{\varphi=v} + \frac{3}{16\pi^2} \left(\frac{\partial M_t^2}{\partial \varphi_2}\right)_{\varphi=v}^2 \left[f''(m_t^2) - f''(M_t^2)\right]_{\varphi=v}, \quad (133)$$

and then, observing that $f''(m^2) = 2 \log(m^2/Q^2)$,

$$(\Delta \mathcal{M}_R^2)_{22} = \frac{3}{8\pi^2} \frac{g^2 M_t^4}{m_W^2 \sin^2 \beta} \log \frac{m_t^2}{M_t^2}. \quad (134)$$

It is now a simple exercise to derive the one-loop-corrected eigenvalues m_h and m_H , as well as the mixing angle α associated with the one-loop-corrected mass matrix (130). The most striking fact in eq. (134) is that the correction $(\Delta \mathcal{M}_R^2)_{22}$ is proportional to (M_t^4/m_W^2) . This implies that the tree-level predictions for m_h and m_H can be badly violated, and so for the related inequalities. The other free parameter in eq. (134) is $m_{\tilde{q}}$, but the dependence on it is much milder.

The phenomenology of the MSSM Higgs bosons has been discussed in some detail in a recent review [89], so we can afford to be very brief here. Supersymmetric Higgs bosons have been intensively searched for at LEP, which in 1997 has collected about 50 pb^{-1} at $\sqrt{s} = 183 \text{ GeV}$. LEP searches are based on two complementary processes: $e^+e^- \rightarrow hZ$, whose cross-section is proportional to $\sin^2(\beta - \alpha)$, and $e^+e^- \rightarrow hA$, whose cross-section is proportional to $\cos^2(\beta - \alpha)$. Taking into account that no significant excesses with respect to the expected background have been reported for the 1997 run, the combination of these two processes should allow to establish, both for h and for A , an absolute lower bound of the order of 75 GeV , for typical values of the parameters controlling the radiative corrections [1]. With the present energy and luminosity, the Tevatron collider is not very sensitive to the MSSM Higgs bosons: the present limits on the charged Higgs mass from top decays [8] are significant only for values of $\tan \beta$ outside the preferred range $1 < \tan \beta < M_t/M_b$. Unfortunately, even by further raising the energy towards $\sqrt{s} = 200 \text{ GeV}$, LEP will not be able to explore completely the parameter space of the MSSM Higgs sector [13]. In the unfortunate case that no Higgs boson is found at LEP, the search for SUSY Higgs bosons will be continued at the LHC. The first LHC studies (see, e.g., [90] and references therein), which focused on the simplified case of heavy supersymmetric particles, have been considerably improved by the computation of the most important MSSM corrections to the relevant production processes, by the inclusion of possible Higgs decays into pairs of lighter supersymmetric particles, and by more accurate experimental simulations (see e.g. [89] and references therein). A complete no-lose theorem is not available, but it seems quite plausible that, if the MSSM is correct, at least part of its Higgs sector will not escape detection at the LHC. A more complete exploration of the MSSM Higgs sector could then be pursued at some high-energy linear e^+e^- collider, of the type currently under study.

5.5.4 Sparticle searches

As should be clear by now, the general framework of supersymmetry is so flexible that it is very difficult to give a unified description of the searches for supersymmetric particles. In the following, we shall briefly review the present bounds (no signal of supersymmetry has been observed yet!) and the future discovery potential, organizing the discussion around the most important machines contributing to these searches. Unless otherwise stated, we shall assume R-parity conservation and work in the case of a heavy gravitino, but here and there we shall also comment on the light gravitino case and on the possibility of broken R-parity. Even with these restrictions, the complex interplay of the dependences of masses, cross-sections and branching ratios on the various parameters makes it very difficult to specify simple general limits. Sometimes, one may choose to combine different searches within the so-called ‘constrained MSSM’: this means assuming universal boundary conditions on the soft masses at M_U so that the low-energy spectrum and interactions are essentially described (modulo some subtleties for the stop sector) by four basic parameters, for example m_0 , $m_{1/2}$, μ and $\tan \beta$.

LEP

LEP1 is still a solid basis for very general limits on the sparticle spectrum. Working on the Z peak, and using both indirect constraints from the line shape and dedicated searches, all conceivable decays of the Z boson into pairs of supersymmetric particles were studied, with high statistics and controllable backgrounds. As a rule of thumb, this allowed to exclude most supersymmetric particles up to mass values of the order of $m_Z/2$: the only possible exceptions were particles with suppressed couplings to the Z boson, such as the lightest neutralino $\tilde{\chi}$ or the lightest stop \tilde{t}_1 , for special choices of the corresponding mixing parameters.

At LEP2, the production cross-sections for sparticle pairs are more model-dependent than at LEP1, but, thanks to the higher energy, much stronger limits could be obtained. For example, chargino pair production is controlled by s-channel (γ, Z) exchange and by t-channel $\tilde{\nu}_e$ exchange, with the possibility of destructive interference in the case of a light sneutrino. Since chargino decays involve the lightest neutralino, the mass difference between the lightest chargino and the lightest neutralino is another important parameter for the searches. Barring special corners of the parameter space with low acceptance (almost degenerate chargino and neutralino) or low cross-section (light sneutrino), and given the absence of a signal over the background, the lower bound on the chargino mass is very close to the kinematical limit. After the 1997 run at $\sqrt{s} \simeq 183$ GeV, the four LEP experiments [91] give bounds above 90 GeV.

Also associated production of neutralinos ($\tilde{\chi}\tilde{\chi}'$), of charged sleptons ($\tilde{l}^+\tilde{l}^-$ and of stop squarks ($\tilde{t}_1\tilde{t}_1$) can be used to obtain interesting limits at LEP2. All these processes occur via s-channel exchange of neutral vector bosons. In the case of selectron production, there is an important additional contribution from t-channel neutralino exchange, which may increase the cross-section substantially. In the constrained MSSM, the combination of chargino and neutralino searches can be used to set a lower bound on the lightest neutralino, but this lower bound has a significant dependence on the minimum allowed values for the sneutrino mass and for $\tan\beta$. Typical limits on the charged sleptons are in the 60-80 GeV region, depending on the slepton flavour and on some model assumptions, such as the allowed amount of mass degeneracy between left and right sleptons, and between sleptons and the lightest neutralino. One of the reasons why the sleptons limits are in general weaker than the chargino limits is the strong p-wave phase space suppression near threshold.

Comparable limits can be derived for the cases of light gravitino and of broken R-parity, when the lightest MSSM particle is allowed to decay.

Hadron colliders

Being strongly interacting sparticles, squarks and gluinos are best searched for at hadron colliders. Both in the heavy and in the light neutralino case, production cross-sections for $\tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q}$ pair-production in pp or $p\bar{p}$ collisions are relatively model-independent functions of $m_{\tilde{g}}$ and $m_{\tilde{q}}$. As far as signatures are concerned, one has to distinguish two main possibilities: if $m_{\tilde{g}} < m_{\tilde{q}}$, then $\tilde{q} \rightarrow q\tilde{g}$ immediately after production, and the final state is determined by \tilde{g} decays; if $m_{\tilde{q}} < m_{\tilde{g}}$, then $\tilde{g} \rightarrow \tilde{q}\bar{q}$ immediately after production, and the final state is determined by \tilde{q} decays. The first case is favoured by the constrained MSSM. In old experimental analyses, it was customary to work under a certain set of assumptions: 1) five or six (\tilde{q}_L, \tilde{q}_R) mass-degenerate squark flavours; 2) LSP $\equiv \tilde{\gamma}$, with mass negligible with respect to $m_{\tilde{q}}, m_{\tilde{g}}$; 3) the dominant decay modes of squarks and gluinos are the direct ones, $\tilde{g} \rightarrow q\bar{q}\tilde{\gamma}$ if $m_{\tilde{g}} < m_{\tilde{q}}$ and $\tilde{q} \rightarrow q\tilde{\gamma}$ if $m_{\tilde{q}} < m_{\tilde{g}}$. The signals to be looked for are then multijet events with a large amount of missing transverse momentum. To derive reliable limits, however, one has to take into account that the above assumptions are in general incorrect. For example, one can have cascade decays $\tilde{g} \rightarrow q\bar{q}\tilde{\chi}_{i \neq 1}^0, q'\bar{q}\tilde{\chi}_k^\pm \rightarrow \dots$ and $\tilde{q} \rightarrow q\tilde{\chi}_{i \neq 1}^0, q'\tilde{\chi}_k^\pm \rightarrow \dots$. The effects of these cascade decays become more and more important as one moves to higher and higher squark and gluino masses. Taking all this into account, the present limits from the Tevatron collider are roughly in the 200 GeV range (for recent

reviews, see e.g. [92]). At the LHC (for recent studies, see e.g. [93]), CMS and ATLAS should be able to explore squark and gluino masses up to 1-2 TeV, essentially filling the MSSM parameter space allowed by theoretical prejudices on naturalness.

The searches for charginos and neutralinos at hadron colliders are not very competitive in the heavy gravitino case. On the other hand, the smaller backgrounds for the final states with hard photons gives hadron colliders an advantage in the light gravitino case. For example, in typical messenger models, the present Tevatron data can be used to rule out [94] neutralinos up to 70 GeV and charginos up to 150 GeV.

5.6 Concluding remarks on supersymmetry

The aim of this long section was to explain, to an audience mainly composed of young experimentalists, why low-energy supersymmetry is a motivated and phenomenologically viable extension of the SM near the electroweak scale, which will be directly tested in the next few years.

The audience should have realized that the phenomenological studies of MSSM signals at present and future accelerators are at an advanced stage, and are continuously improving. Important indirect tests of SUSY are also possible in the realm of flavour physics. Given the present absence of definite experimental or theoretical evidence, in setting up the framework for these searches we should not be prisoner of too restrictive frameworks: Nature may have more imagination than we do!

On the theoretical side, some major open problems remain: the dynamics of SUSY breaking, the SUSY flavour puzzle, the cosmological constant problem. Despite the intense theoretical activity on all of them, the feeling is that some firm guiding principle is needed to make substantial progress. The present hope is that string theories and their fascinating duality properties will provide it, when better understood. The subject is still young, and there is a lot of room left for future investigations ...

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